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ELEMENTARY ARITHMETIC

BY

THOMAS W. PIPER.

1/6

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ELEMENTARY ARITHMETIC

FOR

SCHOOLS AND COLLEGES.

BY

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PREFACE.

THIS little book is intended as an introduction to a more comprehensive study of Arithmetic, being, in fact, designed to introduce the Author's "*Advanced Arithmetic.*"

Being thus intended for beginners, it necessarily differs in many respects from its larger companion—*e.g.*, it deals more exhaustively with the very rudiments of Arithmetic; it often contains an explanation or an illustration instead of a definition; in it the Compound Rules are introduced as early as possible, because, dealing as they do with Concrete numbers, they offer more attractions to young pupils than are presented by numbers purely Abstract; it deals with Multiplication before Subtraction, because the former is based upon the same idea as Addition—viz., *the increasing of numbers*, while an entirely new idea is the basis of Subtraction—viz., *the decreasing of numbers.* It will, however, be observed that the usual order is followed when Decimals are treated of, and this is done because it is expected that by the time a pupil has advanced thus far he will find no difficulty in grasping all that is necessary in order to fully understand the Reason of the process of

Subtraction of Decimals. The fact that the process of Multiplication of Decimals is somewhat more complex than Subtraction of Decimals, furnishes an additional reason why the latter should be treated of before the former. And here it may be remarked that it is not advisable to make children perfect masters of every point in one "Rule" before proceeding to give them *some* knowledge of succeeding ones. For instance, is it necessary that a child should be able to "*Find the value of* 8043 *sheep at £7*, 4s. 10½d. *each*," before he is taught to "*Find the price of a pound of butter, when* 3 *pounds cost* 4s. 6d.?" A more useful method of procedure is to give pupils general ideas of all the leading propositions in Arithmetic by means of oral questions in Mental Arithmetic, to follow more slowly with *easy* questions on slates or paper, and more slowly still with *difficult* questions. Thus we might see a pupil who (1) could solve *mentally* such a problem in **Fractions** as this—*If one-fifth of a piece of timber weighs* 17 *cwt., what is the weight of the whole piece?*" (2) could work on his slate such an *easy* sum in **Compound Division** as the following—"*Five men had a dinner together, and the bill came to* 12s. 6d., *what had each to pay?*" and (3) could work on his slate such a *difficult* sum in **Simple Multiplication** as, "*Find the value of* 603128 × 700980.

Again, for the sake of young teachers, it may be remarked that the two chief uses of very difficult sums are—

1st. To test the *mechanical ability* of the pupils to apply the ordinary rules of Arithmetic to very extreme cases; and,

2nd. To strengthen the *mental faculties* of the pupils by causing them to exercise their powers of mind upon difficult problems.

Hints for teachers have been given here and there throughout the book. These have been inserted for the benefit of young teachers. They are not meant to be *exhaustive*, but must be taken as specimens of methods of teaching. Neither do they pretend to be the *best* methods by which the subjects they deal with could be handled. Their office is simply *to suggest*, and their chief feature is that they aim at making things as clear, as interesting, and therefore as easy, as possible.

With the exception of a few Examples * in Ex. xxviii., all the exercises in this book are new.

T. W. P.

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* These are specially indicated.

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ELEMENTARY ARITHMETIC.

CHAPTER I.

PRELIMINARY.

I. DEFINITIONS, &c.

1. Of the origin of "Number."

A savage untutored in the science of number is often found at a loss to express precisely the number of his foes, the lapse of time, the price of anything he has to sell. In this difficulty he resorts to counting on his fingers. For instance, he observes perhaps that, since he last went forth to war, as many moons have waxed and waned as he has fingers on one hand. To express this interval of time he therefore points to the moon and then tells off the fingers of his hand. In the same way he would express the number of his children, and in counting over his fingers for this last purpose he may happen to notice that he has as many children as fingers on one hand, and that he has also been as many months at peace as he has fingers on one hand. In this way he may acquire an idea of the number FIVE, which idea, vague and indistinct at first, would be confirmed and enlarged by his daily experiences. In process of time it would come about that, whereas at first he only conceived the number FIVE as expressing a certain definite number of birds, fishes, enemies, &c., he would in the end acquire an abstract idea of what is meant by the number FIVE, irrespective of any concrete unit (§ 3); in other words, he would realise FIVE as an abstract number (§ 4).

In the same way he would obtain correct ideas of all other numbers, and of the relative values of numbers, such as that FIVE is less than TEN; that FIVE can twice be taken from TEN; that TWO must FIVE times be taken to make TEN.

2. That *the abstract ideas of number* (represented by the words ONE, TWO, THREE, &c.,) *thus formed, at first expressed by written* WORDS, *are now, for convenience, more generally signified by* FIGURES.

By some such process as we have described above (§ 1) must all ideas requiring comparison and judgment for their acquisition have been obtained, and man being a social creature, thus have been led to feel the need of *words* to convey his ideas to his fellows. Thus it came to pass that the words ONE, TWO, THREE, &c., were adopted to express ideas of number already formed in the mind.

But since power and position and many comforts depended upon the number of cattle, &c., a man possessed, the study of numbers soon became a very important one, the consequence of which was that the written words by which numbers were at first expressed were superseded by shorter SIGNS called FIGURES (*i.e.*, FORMS).

3. Of the number ONE.

It is manifest that the number FIVE contains the number ONE as many times as there are fingers on a man's hand; it may therefore be said to contain a certain number of ONES, and the same may be said of SEVEN, EIGHT, NINE, and every other such number.

Now, *any one thing considered as being whole and undivided is called a* UNIT,* concerning which we may remark—

(1.) That *a Unit is not necessarily an indivisible whole;* but, on the contrary, it is often divisible into other units—*e.g.*, we may say that a school contains eight *classes* where a *class* is the unit; but each class may be divided into two *sections* in which a *section* is the unit, and each section may contain thirty *boys* where a *boy* is the unit; but in every case the unit is "**considered** as being whole and **undivided**."

(2.) *That a Unit may be either one animal, one person, one thing, or the number* ONE *itself.*

When one animal, one person, or one thing is used as a unit, it is called a CONCRETE unit; when the number ONE itself is used as a unit, it is called the ABSTRACT unit.

(3.) *That the number* ONE *itself is called* UNITY.

The term "Unity" is thus equivalent to "ONE UNIT," and expresses the simple value of the figure 1.

---

* Cf. *Unite*, to join in ONE; *Union*, a joining in ONE.

4. Of numbers generally.

We may now explain that—

(1.) *A collection of* ONES *or* UNITS *is called an* INTEGER *or* WHOLE NUMBER.

Thus, the numbers *six, seven, eight* are integers.

(2.) *All numbers must be either—*

(*a.*) CONCRETE—consisting of concrete units; or,

(*b.*) ABSTRACT—consisting of abstract units.

CONCRETE NUMBERS are further defined as being *those of which the nature of the unit is determined;* while ABSTRACT NUMBERS are *those of which the nature of the unit is not determined.*

In the expressions *eight boys, seven girls,* the numbers *eight* and *seven* are called concrete numbers, because they refer to the particular concrete units *one boy, one girl;* but when we speak of the numbers *eight* and *seven* with reference simply to the abstract unit *one,* these numbers are called abstract numbers.

(3.) *That the Science which treats of numbers is called* **Arithmetic.**

(4.) That in Common Arithmetic all numbers are expressed by means of the nine figures 1, 2, 3, 4, 5, 6, 7, 8, 9 (named respectively *one, two, three, four, five, six, seven, eight, nine*), and the figure 0 (called a *cypher* or *zero,* and having no value in itself).

These figures are sometimes called *Digits,* and sometimes the *nine Digits and Zero.*

The word *Digit* is derived from the Latin *digitus,* a finger or toe, probably because the earliest calculations were effected by countings on the fingers or toes (§ 1).

II. NOTATION AND NUMERATION.

(1.) The Arabic System.

5. Of Intrinsic and Local Value.

It has already been explained that all numbers are expressed by means of the ten *Figures* called *the nine Digits and Zero.* The art of so expressing numbers is called **Notation,** and the art of reading numbers expressed by Notation is called **Numeration.**

It has further been explained that the name of each of the nine digits

expresses its value when standing alone. Thus, the value of the digit **5** is expressed by its name FIVE, meaning FIVE UNITS. This value of the digit 5 is called its **Simple** or **Intrinsic Value.**

In order to a correct understanding of Notation it is necessary to understand that each figure has another value depending simply upon its position, and hence called its **Local*** **Value.** For instance, the two digits, 2 and 5, placed close together, thus, **25**, signify TWENTY-FIVE; but when written thus, **52**, they signify FIFTY-TWO. In like manner, a 3 and a 6 written thus, **36**, signify THIRTY-SIX; but when written thus, **63**, they signify SIXTY-THREE. Similarly the expression **79** signifies SEVENTY-NINE, while **97** signifies NINETY-SEVEN. Now, writing down these numbers thus—

25; **52**; **36**; **63**;
TWENTY-**FIVE**; FIFTY-**TWO**; THIRTY-**SIX**; SIXTY-**THREE**;
79; **97**;
SEVENTY-**NINE**; NINETY-**SEVEN**;

We notice—

(1.) That of the two figures used to express each of them *the one on the right retains its own intrinsic value—e.g.*, in **79** [SEVENTY-**NINE**], the value of the digit **9** is the intrinsic value signified by its own name.

(2.) That *the left hand figure has a new and greater value dependent upon its position*, and hence called its local value; and that in each of the above cases *the local value of a figure, when on the left, is* TEN *times as great as its intrinsic value—i.e., its value when on the right—e.g.*, the value of the **9** in **97** is NINETY—*i.e.*, TEN times its value in **79**.

6. Of the art of expressing numbers greater than NINE and less than a HUNDRED.

A further consideration of the facts mentioned in § 5, teaches us that in such numbers as 97, &c., the right hand figure expresses as many UNITS or ONES as are signified by its name, while the left hand figure expresses as many TENS as are signified by its name—thus, in 97, the 7 signifies 7 *units*, and the 9 signifies 9 *tens*. From which it follows that any number less than *a hundred* may be similarly expressed—*e.g.*, THIRTY-FIVE may be written thus, 35.

---

* Cf. *Locality*, a place; *located*, put in a place; *a local newspaper*, a newspaper for a particular place.

Exercise 1.

Express in figures—

Sixty-two; thirty-seven; forty-one; thirty-eight; eighty-five; seventy-four; eighty-three; fifty-five.

7. Of the art of expressing numbers greater than NINETY-NINE and less than a THOUSAND.

We already know that 25 signifies TWENTY-FIVE. It may now be explained that **3**25 is used to signify **THREE HUNDRED** AND TWENTY-FIVE; that **4**25 is used to signify **FOUR HUNDRED** AND TWENTY-FIVE; and that **7**25 is used to signify **SEVEN HUNDRED** AND TWENTY-FIVE; where *the* 3, *the* 4, *and the* 7 *are each used with a local value* ONE HUNDRED *times greater than its intrinsic value.*

Similarly, 444 is used to signify FOUR HUNDRED AND FORTY-FOUR, where

The 4 on the right signifies FOUR—*i.e.*, FOUR **UNITS**;

The 4 in the second place from the right signifies FOUR **TENS**;

The 4 on the left—*i.e.*, in the third place from the right, signifies FOUR **HUNDREDS.**

From which it is evident that, in any integral number—

(1.) The figure on the right expresses as many UNITS as its name implies, and is therefore said to be *in the place of* UNITS.

(2.) The figure in the second place from the right expresses as many TENS as its name implies, and is therefore said to be *in the place of* TENS.

(3.) The figure in the third place from the right expresses as many HUNDREDS as its name implies, and is therefore said to be *in the place of* HUNDREDS.

Or, since *a hundred* is equivalent to *ten tens*, we may say that *the intrinsic value of a digit is increased* TEN FOLD *by moving it one place further to the left.*

From all this it follows that any number less than *a thousand* may be similarly expressed—*e.g.*, FIVE HUNDRED AND THIRTY-SEVEN may be written thus, 537.

Exercise 2.

1. *Express in words—*

327; 462; 891; 723; 536; 955; 438; 649; 888; 999.

2. *Express in figures—*

Four hundred and eighty-six; three hundred and sixty-five; five

hundred and seventy-eight; nine hundred and thirty-one; two hundred and sixty-two; one hundred and twenty-nine; seven hundred and fifty-seven.

8. Of the art of expressing numbers greater than NINE HUNDRED AND NINETY-NINE.

Before proceeding further, it should be explained that *ten hundreds are called one* **thousand**, and that *one thousand thousands are called one* **million**.

It must now be explained that, *in order to read any number expressed by more than three figures, it is usual to arrange the figures in groups of three each*, thus—

31,427,938,

where it will be observed that *the grouping proceeds from the right.*

Now *each of these groups is read as a distinct and independent number* —thus, in the above example—

31 is read *thirty-one;*

427 is read *four hundred and twenty-seven;*

938 is read *nine hundred and thirty-eight;*

but, whereas the

938 is read *nine hundred and thirty-eight* simply, the

427 is read *four hundred and twenty-seven* THOUSAND, and the

31 is read *thirty-one* MILLION;

the whole eight figures being therefore read thus—*Thirty-one million, four hundred and twenty-seven thousand, nine hundred and thirty-eight.*

Similarly the nine figures,

444,444,444,

are read thus—*Four hundred and forty-four million, four hundred and forty-four thousand, four hundred and forty-four;* when it is seen that the value of the digit 4 is uniformly increased TEN times by moving it one place further to the left (§ 7). In the same way the nine digits,

938,938,938,

are read thus—*Nine hundred and thirty-eight million, nine hundred and thirty-eight thousand, nine hundred and thirty-eight;* from which we conclude that, in such numbers—

a. The digits forming the group on the right are always to be read simply;

b. The digits forming the second group from the right are to be read as *so many thousands;* and

c. The digits forming the next group are to be read as *so many millions.*

9. The complete system of Notation will now be evident from the following table :—

MILLIONS.				THOUSANDS.						
Hundreds.	Tens.	Units.	,	Hundreds.	Tens.	Units.	,	Hundreds.	Tens.	Units.
3	2	7	,	4	3	6	,	8	5	1

N.B.—It should be carefully noticed that such a number as 45873 may be read in several ways, thus—

(1.) 45 *thousands*, 8 *hundreds*, 7 *tens*, 3 *units;*
(2.) 458 *hundreds*, 7 *tens*, 3 *units.*
(3.) 4587 *tens*, 3 *units.*
(4.) 45873 *units.*

Exercise 3.

1. *Express in words—*

27,426 ; 38,219 ; 532,746 ; 281,179 ; 58,623 ; 73,285 ; 2,341,252 ; 5,726,839 ; 63,247,826 ; 391,233,474 ; 85,697,333.

2. *Express in figures—*

Five thousand four hundred and sixty-two ; seven thousand eight hundred and thirty-one ; eighty-seven thousand four hundred and fifty-two ; six thousand six hundred and fifty-one ; nine hundred and forty-one thousand six hundred and twenty-four ; forty-four thousand eight hundred and ninety-nine ; two million five hundred and fifty-five thousand three hundred and sixty-six ; five hundred and eighty-three million seven hundred and seventy-one thousand two hundred and twenty-two ; eight hundred and eighty-eight million seven hundred and seventy-seven thousand three hundred and thirty-three.

10. Of some difficulties in Notation.

(1.) We have shown how to express such a number as TWENTY-FIVE, but have omitted to show how such numbers as TEN, TWENTY, &c., &c., may be expressed.

It has been established that *the value of any digit is increased* TEN-*fold by moving it one place further to the left;* now, the value of the digit **1**,

standing in the first place on the right, we know to be ONE, in the second place from the right it would therefore be ONE TEN—*i.e.*, TEN; we therefore express TEN by writing the digit **1** followed by zero, thus—**10.**

Similarly, TWENTY is expressed thus, **20**
THIRTY ,, ,, **30**
FORTY ,, ,, **40**
FIFTY ,, ,, **50**, &c.

Therefore, such a number as EIGHT HUNDRED AND FORTY would be expressed thus—**840.**

(2.) We have now shown how to express TEN and TWENTY, but not the intermediate numbers ELEVEN, TWELVE, THIRTEEN, &c.

Since ELEVEN is equivalent to ONE more than TEN, it is written thus, **11**; for a similar reason

TWELVE is expressed thus, **12**
THIRTEEN ,, ,, **13**
FOURTEEN ,, ,, **14**, &c.

Therefore, such a number as SEVEN HUNDRED AND FIFTEEN would be expressed thus—**715.**

(3.) We have also shown how to express such a number as FOUR HUNDRED AND TWENTY-FIVE, but not such a number as FOUR HUNDRED AND FIVE.

Let it be considered that, if the number to be written had been *four hundred and* SEVENTY-*five*, it would have been expressed thus, 475; that *four hundred and* NINETY-*five* would have been expressed thus, 495; but that, as in the given number, *four hundred and five*, nothing is said of *seventy*, *ninety*, or any other number of TENS, no digit can be put in the "*place of tens.*" Since, however, the digit 4 is required to signify *four hundreds*, it must be in the "*place of hundreds—i.e.*, in the third place counting from the right; it is therefore essential that *some* figure should come between it and the digit 5; the figure **0** is therefore placed between them, and the given number is expressed thus—**405.**

In all cases where the number to be written does not include digits in all the possible places, the following rule should be followed:—*Put a cypher in every place in which a digit might have been placed.*

According to this rule,

Four hundred is expressed by 400
Six thousand and forty seven is expressed by 6,047
Eight thousand and one ,, 8,001
Forty million and thirty ,, 40,000,030
&c. &c.

Exercise 4.

1. *Express in words—*

17; 26; 18; 80; 403; 1709; 63080; 54600; 3090; 7902; 280601; 700061; 2080639; 50200701; 180601017; 500001006; 301000090; 820016700.

2. *Express in figures—*

Nineteen; seventy; six hundred and five; three thousand eight hundred and two; ninety thousand six hundred and fifty; seventeen thousand one hundred; two hundred thousand and two; five million sixteen thousand one hundred and one; twenty-six million and nineteen; one hundred million two thousand six hundred.

(2.) The Roman Notation.

11. The system of notation, of which the above is a brief description, will more fully be described when we come to the consideration of Decimal Fractions. It was introduced into Europe by the Arabs, who had obtained it from the Hindoos. It is therefore known as the *Arabic System*, and is the one in common use.

Another and more cumbrous system was in use among the Romans, of which the following is a brief description:—

1st. *Instead of Figures being used to express numbers, the following letters were employed*—viz., I, V, X, L, C, D, M, of which the simple values were respectively 1, 5, 10, 50, 100, 500, 1000.

Five hundred was primarily represented by IↃ, which was afterwards contracted to D. CIↃ was also employed to represent *one thousand*.

2nd. It will be noticed that, whereas in the Arabic system the simple value of no digit is greater than NINE, in the Roman system single letters represented such high numbers as FIVE HUNDRED and ONE THOUSAND.

3rd. Each of the above *seven letters* always represented the same number, *whatever its position might be*, thus differing from the manner in which the ten figures are used in the Arabic system—*e.g.*, in the latter system the figure **9** standing by itself signifies NINE, but in the expression **94** it denotes NINETY; but no *general* principle of this kind obtained in the Roman system.

The difference between the principles upon which these two systems

are founded may be expressed by saying that, *whereas in the Arabic system high numbers are expressed by substituting local values for the simple values of the digits, and* ADDING *thereto the simple value of a digit if required—*

Thus, to express NINETY-FOUR, the simple value of the digit **9** is increased to *ninety*, and the simple value of the digit **4** is added, thus, **94**.

In the Roman system numbers were expressed by single letters, the values of which were INCREASED *or* DIMINISHED *according as it was most convenient*, thus—

(1.) *If two characters of the same value were placed side by side*, or, *if a character was followed by one of less value than itself*, the number denoted by the expression was the SUM (§ 16) of their simple values—thus, XX signifies 20; XI signifies 11.

(2.) *If a character was followed by one of greater value than itself*, the number denoted by the expression was the DIFFERENCE (§ 16) of their simple values, thus, IX signifies 9; XL signifies 40.

4th. But although in the Roman system there was no general law upon which the local value of a letter depended, there were certain cases where the same principle was recognised, for

(1.) *The value of Iↀ was increased ten-fold by every ↀ annexed to it;* thus 5000 was indicated by Iↀↀ.

This is very similar to the Arabic method of making the value of a digit ten times greater by writing **0** after it.

(2.) In the same way, *the value of CIↀ was increased ten-fold by adding to it a C and a ↀ;* thus 10,000 was denoted by CCIↀↀ.

N.B.—It should here be noticed that whereas in the Arabic system the local value of a digit is only increased by writing figures *after* it (*i.e.*, on the *right* of it), the value of CIↀ was in the Roman system increased by characters placed both *before and after it.*

(3.) The value of any character was increased a thousand-fold by drawing a line over it, thus $\overline{\text{V}}$ signifies 5000.

Exercise 5.

1. Write in Arabic figures the numbers expressed by XX, XV, XVII, XXVIII, LV, LVII, LXVI, CLX, CCX, DCX, CCC, XL, XIX, XCIX, $\overline{\text{V}}$VI, $\overline{\text{CLI}}$DLXII.

2. Express in Roman Notation the numbers 30, 70, 12, 15, 18, 27, 36, 51, 108, 63, 220, 4017, 63900, 170108, 56103, 184907.

III. USES OF TERMS AND SIGNS IN ARITHMETIC, &c., AND THE NATURE OF ITS ELEMENTARY OPERATIONS.

12. That Numbers may be Increased and Diminished.

A moment's consideration is sufficient to show that numbers may be (1) *Increased* and (2) *Diminished*, *e.g.*, a boy may have 9 marbles in his pocket. If 5 more be given him he will then have 14 marbles. Here the concrete number 9 *marbles* would have been *increased*. If, however, instead of having 5 marbles given him he had *lost* 5 marbles, he would have had but 4 marbles remaining. In this case the concrete number 9 *marbles* would have been *diminished*.

13. Of the Increasing of Numbers.

Now, a number may be increased in two ways, which will be best seen from an example—

Ex. *If there are 3 baskets, each basket containing 6 oranges, how many oranges are contained in the 3 baskets?*

We have given us the number of oranges in one basket, viz., **6**. Now, this number, **6**, is to be so *increased* as to express the number of oranges in all the baskets taken together.

This may be done in two ways—

1st. We may consider thus, **6** *increased by* **6** *increased by* **6** amounts to **18**. Therefore, the total number of oranges is 18.

But in practice we never write *increased by*, but we use the sign $+$, which is called *plus*,* and signifies that the number *after* which it stands is to be increased by the number *before* which it stands, so that the expression—

6 *increased by* 6 *increased by* 6,

might be written thus—

$$6 + 6 + 6.$$

---

* Cf. **Plural**, *more than one*; **pluralist**, a clergyman who holds *more than one* ecclesiastical benefice.

Or, 2d. We may consider thus:—*There are altogether* 3 SIXES *of oranges;* now 3 SIXES of any units we know (by a Table given on page 25) amounts to 18 of those units, so that we say *the total number of oranges is* 3 SIXES *of oranges, i.e.,* 18 *oranges.*

In this case the SIX is made 3 *times greater*, and in practice the expression—

6 *made* 3 *times greater*

is written thus—

$$6 \times 3.$$

The sign (×) is called *Into*, and when placed between two numbers, signifies that the number *after* which it stands is to be taken as many times as are expressed by the number before which it stands.

Now, when a number is increased by the operation signified by the sign "*plus*," it is said to be increased by the method of "**Addition**," but when the operation indicated by the sign "*into*" is used, the number is said to be increased by the method of "**Multiplication**."

14. Of the Diminishing of Numbers.

A number may also be diminished in two ways, as will also be best seen from an example.

Ex. *A certain number of boxes contained* 5 *oranges in each box; if there were altogether* 20 *oranges, how many boxes were there?*

We have here to reduce the number **20**, which expresses the total number of oranges, until it is small enough to express the number of boxes in which those oranges were contained.

Now we may do this in two ways.

1st. We may reduce 20 by successive FIVES until NONE remain, and then counting the number of FIVES thus taken from 20, find the number required, which will be **4**.

The operation, fully described, proceeds thus—

20 *reduced by*	5	leaves	15	as a remainder.	
15	,,	5	,,	10	,,
10	,,	5	,,	5	,,
5	,,	5	,,	0	,,

It is clear that we have *taken away* 4 FIVES *from* 20, it is therefore clear that *in* 20 *there must be* 4 FIVES; therefore the Answer to the Question must be "**4 boxes**."

In practice, the expression—

20 *reduced by* 5, or, 20 *diminished by* 5,

would be written thus—

$$20 - 5.$$

The sign (—) is called *Minus*, and when placed between two numbers signifies that the number *after* which it stands is to be diminished by the number *before* which it stands, OR, that the number *following* it is to be taken from the number *preceding* it.

Or, 2d. We may consider that, seeing (by the Table on page 25) we require 4 FIVES to make TWENTY, there must have been 4 boxes, each containing 5 oranges, if there were 20 oranges altogether.

In this case the number TWENTY is made 5 *times less*, and in practice the expression—

20 *made* 5 *times less* (*more properly*, "*reduced to one-fifth of itself*,")
is written thus—
$20 \div 5.$

The sign (÷) is called *By*, and when placed between two numbers signifies that it is required to find how many times the number *after* which it is placed contains the number *before* which it is placed.

Now, a number diminished by the operation indicated by the sign "*Minus*," is said to be diminished by the method of "**Subtraction**," but when reduced by the method signified by the sign "*By*," it is said to be reduced by the method of "**Division**."

If 6 be increased by 3, and 11 be diminished by 2, the results are the same; therefore we may write—

$$6 + 3 \text{ is equal to } 11 - 2$$

which we generally write thus—

$$6 + 3 = 11 - 2.$$

The sign (=) is called *the sign of equality*, "*equal*," or "*equal to*."

Also, instead of writing the word "*therefore*," we, in Arithmetic, use the sign $\therefore$.

15. Of the Use of Brackets.

Brackets which are of several kinds—*e.g.*, ( ), { }, [ ]—are used to denote that all numbers included within any pair of them are to be con-

sidered as forming but one number, and are therefore to be equally affected by any number not included within the same pair of brackets, thus—

$$(6 + 3) \times 8 = 9 \times 8 = 72.$$

Also, $[8 + 2\{9 + 3(4 + 3) + 17\} + 21] \times 9$

$$= [8 + 2\{9 + 21 + 17\} + 21] \times 9$$
$$= [8 + 94 + 21] \times 9$$
$$= 123 \times 9$$
$$= 1107.$$

A **Vinculum** is a sign sometimes used instead of brackets. It consists of a line drawn over the numbers to be considered as forming one number —thus, $2 \times \overline{8 + 3} = 2 \times 11 = 22$.

CHAPTER II.

ELEMENTARY RULES AND COMPOUND RULES (MONEY).

16. Of the objects of the four Elementary Operations.

(1.) When one number is *added* to another number, the resulting number is called their **SUM**,* thus—

(3 *added to* 8, *i.e.*) $8 + 3 = 11$;

therefore 11 is the *Sum* of 8 and 3.

N.B.—The same holds with more than two numbers, thus—

$$4 + 2 + 3 + 5 = 14,$$

where 14 is the *Sum* of 4, 2, 3, and 5.

Therefore, concisely expressed, the **Object of Addition** may be said to be *to find the Sum of two or more given numbers.*

(2.) When one number is *multiplied* by another number, the resulting number is called their **PRODUCT**;† thus—

(8 *multiplied by* 3, *i.e.*) $8 \times 3 = 24$;

therefore 24 is the *Product* of 8 and 3.

---

* Cf. "*A sum of money*"; the "*summing up*" of a judge.
† Cf. "The animal and vegetable *products* of a country.

N.B.—When more than two numbers are multiplied together, the resulting number is called their *Continued Product*, thus—

$$7 \times 2 \times 4 = 56,$$

where 56 is the *Continued Product* of 7, 2, and 4.

Therefore, concisely expressed, the **Object of Multiplication** may be said to be *to find the Product, or Continued Product, of two or more given numbers.*

N.B.—1. Multiplication is also sometimes signified by placing a dot between the numbers to be multiplied together, thus—

$6 \times 4 \times 3$ might be expressed thus, 6.4.3.

N.B.—2. *The number to be multiplied* is called the "**Multiplicand,**" and *the number by which it is to be multiplied* is called the "**Multiplier;**" thus, in 12×3, the *Multiplicand* is 12, and the *Multiplier* is 3.

(3.) When one number is *subtracted* from another number, the resulting number is called their DIFFERENCE, thus,

(3 *subtracted from* 8, *i.e.*) $8 - 3 = 5$,

where 5 is the *Difference* between 8 and 3.

N.B.—The number taken away is called the *Subtrahend*, and the number from which it is taken is called the *Minuend.*

Therefore, the **Object of Subtraction** may be said to be *to find the difference between two given numbers;* or, *to find what number must be added to one given number to make it equal to another given number;* or, *to find by how many one given number exceeds another given number.*

(4.) When one number is *divided* by another number, the resulting number is called the QUOTIENT.*

The Quotient is thus the number which marks how many times one number is contained in another.

Thus—

(12 *divided by* 3, *i.e.*) $12 \div 3 = 4$,

where 4 is the *Quotient* found by dividing 12 by 3.

N.B.—1. Division is also sometimes signified by writing one number upon the other, thus—

$12 \div 3$ may be written $\frac{12}{3}$.

N.B.—2. *The number to be divided* is called the "**Dividend;**" and *the number by which it is to be divided* is called the "**Divisor,**" thus—

In $12 \div 3$ the *Dividend* is 12, and the *Divisor* is 3.

---

* Cf. *Quota*, literally, the *how much*, or *how many:* also, *Quote*, literally, to mark *how much.*

Therefore, the **Object of Division** may be said to be *to find the Quotient to two given numbers;* or, *to find how many times one given number contains another given number.*

I. Addition.

17. Of the two kinds of Addition.

Addition is said to be of two kinds, viz. :—

(1.) **Simple Addition**, in which the numbers to be added together are either all abstract numbers (§ 4), or all concrete numbers of the same denomination—as all *pounds*, all *yards*, all *inches*, &c.

(2.) **Compound Addition**, in which the numbers to be added together are all concrete numbers of the same kind, but not expressed in one denomination—as when one sum of money expressed in *pounds, shillings*, and *pence* is to be added to another sum of money also expressed in *pounds, shillings*, and *pence.*

18. Of the Mental Operation of Addition, leading to the Rule for Simple Addition.

In § 1, we have endeavoured to show how ideas of number were first formed in the mind, and how the *qualities* or *powers* of numbers were discovered. Let us now more particularly consider this latter subject in its connection with the *increasing of numbers by means of Addition.*

A person having EIGHT fishes catches ONE more, and finds he then has NINE; he thus learns that *the value of a number increased by* ONE *is expressed by counting* ONE *forward from the original number.*

Again, the same person catching THREE fish one day and TWO the next, finds he has altogether FIVE fishes; he thus learns that *any number increased by* TWO *produces a resulting number found by counting* TWO *forward from the original number.*

In some such ways as these he finds values for the sums of the values of any two of the digits taken together, so that ultimately, without reference to any concrete units, he learns that, in common language, *six and two make eight, seven and five make twelve*, &c. He is now supplied with all the preliminary information necessary for enabling him to enter

upon the study of the arithmetical process called "**Addition.**" * He is, in fact, ready for the following sum :—

Ex. 1. *Add together* SIX *and* EIGHT, *i.e.*,
Find the value of 6 + 8.

He reasons thus—SIX *increased by* EIGHT = FOURTEEN.
Answer, **14.**

But he is a little staggered at this next sum—

Ex. 2. *Add together* FIVE, NINE, *and* SEVEN, *i.e.*,
Find the value of 5 + 9 + 7.

He begins thus—5 + 9 = 14;
But now comes the difficulty, 14 + 7 = *How many?*
He may consider that 14 = 10 + 4,
Therefore, 14 + 7 = 10 + 4 + 7;
And, 4 + 7 = 11,
Therefore, 14 + 7 = 10 + 11;
But 11 = 10 + 1,
Therefore, 14 + 7 = 10 + 10 + 1 = 2 TENS + 1 = 21, *Answer.*

Now, looking back over his working, he may notice that he added the 7 to 4 (*i.e.*, *Units* to *Units*), which gave him 11; that he put down the *units* figure of this Answer (*i.e.*, **1**), and having added its *tens* figure (also **1**) to the *tens* figure in **1**4 (likewise **1**), he thus obtained the figure **2** in the Answer.

Let us watch him with another sum :—

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* *N.B.*—The teacher would do well to present to his pupils a great number of such problems as the two here given. The *abstract truth* involved in each problem might be stated in symbols on the blackboard to fix it in the minds of the children, thus appealing to the *eyes* of the children, *e.g.* (1), *If a boy has* 3 *apples in one pocket and* 2 *in the other, how many has he in all?* Answer, 5 *apples;* hence, therefore 3 + 2 = 5. (2), *If a man had* 3 *black horses and* 2 *grey ones, he would have* (?) . Children reply, "FIVE." (3), *If one army contained* 3 HUNDRED *men, and another contained* 2 HUNDRED *men, how many men were there in all?* Answer, FIVE *hundred.*

In this way a teacher might impart to his pupils an accurate knowledge of the *Table of Addition* without the tedious process of learning it *by rote.* The *Table of Multiplication* makes much greater demands on the memory, for which reason it should be learnt *by heart* after being illustrated and compiled before the children in the same manner as is here recommended for the Addition Table.

Ex. 3. *Add together* FORTY-THREE *and* TWENTY-FIVE, *i.e.*,
Find the value of 43 + 25.

As in Example 2, he considers thus—

3 + **5** = 8,
Therefore, **4**3 + 5 = 48;
And therefore, 43 + **2**5 = 48 + **20**;
But here's the difficulty, *how is* 48 *to be increased by* 20?
Well, 20 = 2 TENS, and 48 = 4 TENS + 8,
Therefore, 48 + 20 = 6 TENS + 8 = **68**, *Answer.*

Now, looking back, we see that the **3** was added to the **5** (*i.e.*, *Units* to *Units*, as in Ex. 2), and **4** was added to the **2** (*i.e.*, *Tens* to *Tens*).

Ex. 4. *Add together* FORTY-SIX *and* EIGHTY-NINE, *i.e.*,
Find the value of 46 + 89.

As in Ex. 2—

6 units + 9 units = 15 units; set down the **5** as *units*, and add the **1** to the *tens*.

As in Ex. 3—

4 *tens* + 8 *tens* = 12 *tens*,

and, adding the **1** *ten* brought from the sum of the units, we have **13** *tens*; set down **13** as *tens*. (§ 9, *N.B.*)

∴ 46 + 89 = **135**, *Answer.*

Ex. 5. *Add together* 27, 53, *and* 68, *i.e.*,
Find the value of 27 + 53 + 68.

Twenty-seven = 2 tens + 7 units,
Fifty-three = 5 tens + 3 units,
Sixty-eight = 6 tens + 8 units.

∴ *Twenty-seven* + *Fifty-three* + *Sixty-eight* =
(2 + 5 + 6) tens + (7 + 3 + 8) units =
13 tens + 18 units =
13 tens + (1 ten + 8 units) =
14 tens + 8 units =
148, *Answer.*

From a consideration of all these Examples, we derive the following

RULE FOR SIMPLE ADDITION.—Having expressed in figures the numbers to be added together, proceed as follows:—

1. Arrange them one beneath the other in such a manner that their units' figures may be in an upright row, and their remaining figures in rows parallel to the row of units, thus—

$$\begin{array}{r} 439 \\ 62 \\ 8341 \\ 796 \\ \hline 9638 \end{array}$$

2. Add up the row of units, express their sum in figures, and put the units' figure of this sum beneath the row of units.

3. Take the remaining figure (or figures) of this sum, and add it to the sum of the figures in the row of tens.

4. Take the units' figure of this new sum, and write it beneath the row of tens.

5. Take the remaining figure (or figures) of this sum, and add it to the sum of the figures in the row of hundreds, and proceed in a manner similar to that followed with the sum of the tens.

6. Repeat this process continually till the row of figures on the extreme left is reached, when this difference must be observed in dealing with the sum of the figures in this row, that, ***instead of setting down the units' figure only, set down all the figures which express this sum.***

The Rule may be illustrated by an Example, thus—

Ex. 6. *Find the value of* 439 + 62 + 8341 + 796.

The sum of the units is 18, *i.e.*, 1 *ten* + 8 *units.*

Set down **8** under the row of *units.*

The sum of the tens is 22, which, with the 1 *ten* carried from the row of *units*, makes 23 *tens*, which is equal to 2 *hundreds* + 3 *tens.*

Set down **3** under the row of *tens.*

The sum of the hundreds is 14, which, with the 2 *hundreds* carried from the row of *tens*, makes 16 *hundreds*, which is equal to 1 *thousand* + 6 *hundreds.*

Set down **6** under the row of *hundreds.*

The sum of the thousands is 8, which, with the 1 *thousand* carried from the row of *hundreds*, make 9 *thousands.*

Set down **9** under the row of *thousands.*

Answer, 9638.

Exercise 6.

Exercises in Simple Addition.

(1.)	(2.)	(3.)	(4.)
27	51	234	518
93	28	609	394
46	75	782	672

(5.)	(6.)	(7.)	(8.)
9201	8017	25006	308210
4678	9608	70090	267048
8432	2990	38460	853906
6785	3086	29909	782680

(9.)	(10.)	(11.)	(12.)
5603	73014	306012	500040
840	6920	73909	600903
907	855	8020	728
6087	619	6080	864
8509	7080	405	19060
63	82401	136720	705602

13. Add together 6007, 82001, 340, 79, 900603, 2708004.

14. Find the sum of 45000601, 38004, 2030006, 79001009, 4080, 79602, 8100706.

19. Extension of the Rule for Simple Addition to find the Rule for Compound Addition.

Ex. 1. *Find a sum of money equal to the sum of* 4 *pence,* 6 *pence, and* 5 *pence.*

By Simple Addition we find that *the sum of the pence is* (4 + 6 + 5) pence, *i.e.*, 15 pence.

But here we must remark that *for* 12 pennies *we can get* 1 silver shilling.*

* The teacher should here produce 12 *pennies* in one hand and *a shilling* in the other, telling the children that the *one shilling* would purchase as much bread as the *twelve pennies.*

Now we know that 15 *is* 3 *more than* 12, therefore 15 *pence is* 3 *pence more than* 1 *shilling, i.e.,*

15 pence = 1 shilling and 3 pence,

which is usually thus written.

1s. 3d., *Answer.*

N.B.—The pupil must here learn to remember that—

4 *farthings*	= 1 *penny*	(1d.)
12 *pence*	= 1 *shilling*	(1s.)
20 *shillings*	= 1 pound	(£1.)
21 *shillings*	= 1 guinea.	
2 *shillings*	= 1 *florin*	(fl.)

£, s. d. are abbreviated forms of the Latin words *libra, solidus, denarius,* the names of certain Roman coins.

Also, 1 *farthing* is written thus, 1 f., or ¼d.; 2 *farthings* thus, 2 f., or ½d.; 3 *farthings* thus, 3 f., or ¾d.

Ex. 2. *Add together* 3*s.* 8*d. and* 6*s.* 7*d.*

Here 8d. + 7d. = 15d. = 1s. 3d. (as in Ex. 1.)
and 3s. + 6s. = 9s.
∴ *Answer* = 9s. + 1s. 3d. = 10s. 3d.

Here it will be noticed that we

1st. Found the sum of the pence (15d.), expressed it in s. and d. (1s. 3d.), set down the pence (3d.), and carried the shilling.

2d. Found the sum of the shillings (9s.), increased it by 1 (for the shilling carried), and set down the total (10s.)

Ex. 3. *Find the sum of* 7*s.* 6*d.,* 8*s.* 5*d., and* 9*s.* 10*d.*

Here the sum of the Pence = 6d. + 5d. + 10d. = 21d. = 1s. 9d.
The sum of the Shillings = 7s. + 8s. + 9s. = 24s.
∴ Answer = 24s. + 1s. 9d. = 25s. 9d. = £1, 5s. 9d.

Ex. 4. *What sum of money is equal to* £3, 4*s.* 7*d.* + £6, 9*s.* 5*d.* + £4, 8*s.* 6*d.* + £5, 3*s.* 2*d.?*

Here the sum of the **Pence** = 7d. + 5d. + 6d. + 2d. = 20d. = 1s. 8d.
The sum of the **Shillings** = 4s. + 9s. + 8s. + 3s. = 24s. = £1, 4s.
The sum of the **Pounds** = £3 + £6 + £4 + £5 = £18.

∴ *Answer* = £18 + £1, 4s. + 1s. 8d.

Where the sum of the **Pence** is 8d.
The sum of the **Shillings** = 4s. + 1s. = 5s.
The sum of the **Pounds** = £18 + £1 = £19.

∴ **Answer, £19, 5s. 8d.**

In Examples 3 and 4 it will be seen that we have proceeded by a method of which a description is contained in the following—

RULE FOR COMPOUND ADDITION OF MONEY.—Having arranged the given sums of money so that the farthings, the pence, the shillings, and the pounds form each a separate vertical column, find the sum of the Farthings, express it in pence and farthings, set down the Farthings and carry the Pence. Next, find the sum of the Pence, increase it by the number of Pence carried from the Farthings, express this last sum as shillings and pence, set down the Pence and carry the Shillings. In a similar way, find the sum of ALL the Shillings, express it in Pounds and Shillings, set down the Shillings and carry the Pounds. Lastly, find the sum of ALL the Pounds, and set it down as Pounds.

We will now illustrate this Rule by an Example.

Ex. 5. *Add together* £7, 19*s.* 6½*d.*, £13, 6*s.* 2¼*d.*, £104, 0*s.* 10¾*d.*, 19*s.* 0½*d.*

Here we have—

(2 + 2 + 3 + 2) f. = 9 f. = 2¼d.
∴ Set down ¼ and carry 2.
(6 + 2 + 10 + 2 carried) d. = 20d. = 1s. 8d.
∴ Set down **8** and carry 1.
(19 + 6 + 19 + 1 carried) s. = 45s. = £2, 5s.
∴ Set down **5** and carry 2.
(7 + 13 + 104 + 2 carried) £ = £126
∴ Set down **126.**
Answer, £126, 5s. 8¼d

£	s.	d.
7	19	6½
13	6	2¼
104	0	10¾
	19	0½
£126	**5**	**8¼**

Exercise 7.*

Exercises in Compound Addition (Money).

1. Find the sum of—

(*a.*) 6d., 3d., 8d., and 4d.
(*b.*) 3d., 9d., 7d., 8d., 5d.
(*c.*) 1s. 3d., 2s. 6d., 5s. 4d.
(*d.*) 3s. 6d., 8s. 5d., 7s. 2d., 13s. 6d.
(*e.*) 6½d., 8¾d., 5¼d., 9¼d., 10½d.
(*f.*) 2s. 0¼d., 6s. 2½d., 3s. 10¾d., 10s. 8½d., 5s. 1¼d.
(*g.*) £3, 6s. 8d., £7, 9s. 6d., £10, 13s. 2d., £11, 4s. 9d.
(*h.*) £8, 2s. 2½d., £6, 4s. 9½d., £2, 8s. 1¼d., £13, 6s. 7¾d.
(*i.*) £9, 2s. 2¾d., £21, 3s. 9¾d., £301, 10s. 8½d., £17, 0s. 4¼d., 18s. 0¾d.

2. If a grocer takes £2, 18s. 7½d. on Monday, and £5, 3s. 0¼d. more than that on Tuesday, how much did he take on the Tuesday?

3. If a horse be worth £50, 10s. 6d., and his carriage be valued at £20, 9s., what is the value of the horse and carriage together?

4. What is the price of a group of three houses if the first be worth £120, 15s., the second £201, 10s., and the third £189, 12s.?

5. John has £20, 7s. 3d., Richard has £50, 6s. 1d.; if Thomas has £40 more than John and Richard together, how much money has he? and how much have all three together?

6. At a sale the pictures were sold for £1040, 12s. 6d., the books for £81, 10s. 9d., the furniture for £601, 3s. 7d., and the carriages for £700. The horses fetched £60, 10s. 6d. more than the pictures, and the house was sold for as much as the horses, carriages, and furniture together. What was the sum of money received for the whole?

---

* Before putting this exercise before his pupils, the teacher should (1) exercise them with such sums as, *How many oranges at a* 1*d. each can be bought for* 6*d.? How many for a* 1*s.? How many for* 1*s.* 8*d.? How many for* 2*s.? How many for* 2*s.* 6*d.?* &c. (2) Reverse these sums, thus, What is the price of 6 *pears at* 1*d. each? at* 2*d. each? of* 20 *at* 1*d. each? of* 30 *at* 1*d. each? of* 30 *at* 2*d. each?* (3) Find from the children's answers that—

20 pence = 1s. 8d.
24 pence = 2s.
30 pence = 2s. 6d.
36 pence = 3s.
&c. &c.

This should be written on the Black Board, copied down, and learn by heart for Home Work. In some such way should all Tables be taught.

II. Multiplication.

20. Of the Mental Operation of Multiplication, leading to the Multiplication Table.

We have already shown how the sum of *any* given numbers may be found by Addition. It may sometimes happen, however,

(1.) *That the numbers whose sum is to be found are all of an uniform value; and*

(2.) *That there are* MANY *of these numbers to be added together.*

For example, it may be required to know *how many pence would pay for* 100 *books at* 5 *pence each.*

Now, it would be a tedious process to set down *one hundred* FIVES and find their sum by the operation of Addition. We should prefer to reason thus—

The price of 100 *books at* 1d. *each would be* ONE *hundred pence.*

∴ *The price of* 100 *books at* 5d. *each would be* FIVE *hundred pence.*

Answer, **500** pence.

Here we consider that *since the price of* ONE *book at* 5d. *was* 5 *times the price of that book at* 1d., *the price of* ONE HUNDRED *books at* 5d. *must in like manner be* 5 *times their price at* 1d.

For which reason

We **multiplied** *the* ONE *hundred pence by* 5, *i.e.,*
We made the ONE *hundred pence five times greater.*

From a consideration of this illustration * we gather that **Multiplication is simply a shortened form of Addition.**

Ex. 1. *Find the sum of* 463 + 463 + 463 + 463.

Proceeding by way of Addition, we find that

(1.) We have in the UNITS column 4 THREES, whose sum is found to be 12. We therefore *put down* **2** *and carry* 1.

(2.) We have in the TENS column 4 SIXES, whose sum is found to be 24, to which we add the 1 carried from the Units. Thus obtaining the number 25, *we put down* **5** *and carry* 2.

(3.) We have in the HUNDREDS column 4 FOURS, whose sum is found to be 16, to which we add the 2 carried from the Tens. Thus obtaining the number 18, we put down **18.**

```
 463
 463
 463
 463
----
1852
```

Answer, **1852.**

* The teacher would, of course, call the attention of his children to *many* such illustrations.

Now, supposing that without going through the process of Addition we had some means of knowing the sum of any number taken four times, it is clear we should much more easily and rapidly find the answer in the above example. This information is contained in the following Table, called

The Multiplication Table.

1	**2**	3	4	5	6	7	8	9	10	11	12
2	4	**6**	8	10	12	14	16	18	20	22	24
3	**6**	9	**12**	15	18	21	24	27	30	33	36
4	8	**12**	16	**20**	24	28	32	36	40	44	48
5	10	15	**20**	25	**30**	35	40	45	50	55	60
6	12	18	24	**30**	36	**42**	48	54	60	66	72
7	14	21	28	35	**42**	49	**56**	63	70	77	84
8	16	24	32	40	48	**56**	64	**72**	80	88	96
9	18	27	36	45	54	63	**72**	81	**90**	99	108
10	20	30	40	50	60	70	80	**90**	100	**110**	120
11	22	33	44	55	66	77	88	99	**110**	121	**132**
12	24	36	48	60	72	84	96	108	120	**132**	144

The manner of using this Table will be best shown by an example, thus—

To find the product of 7 × 8, take the horizontal column which has 7 in its left-hand square, and find what number stands in the square at which

this column intersects the vertical column which has 8 in its top square. The square at the intersection contains the product—viz., 56.

N.B. This Table should be learnt by heart; and it should be noticed that the product of any two numbers is the same whichever of them is taken as the multiplier—*e.g.*, 7 *times* 8 = 8 *times* 7; 3 *times* 11 = 11 *times* 3, &c. This is illustrated in the above Table by a double row of dark figures forming the diagonal of the square.

21. Of the two kinds of Multiplication.

Corresponding to the two kinds of Addition we have also two kinds of Multiplication, viz.,

1. **Simple Multiplication,** where the multiplicand (§ 16, N.B. 2) is either an abstract number or a concrete number expressed in terms of one denomination only.

2. **Compound Multiplication,** where the multiplicand is a concrete number expressed in terms of more than one denomination.

22. Of the Use of the Multiplication Table, leading to the Rule for Simple Multiplication.

Simple Multiplication is generally said to be of two kinds, viz., SHORT MULTIPLICATION and LONG MULTIPLICATION.

I. **Short Multiplication** *is the name given to the process when the multiplier is not greater than* 12. It is so called because in this case we write down the Answer figure by figure at once.

Ex. 1. *Find the sum of* 3142 + 3142 + 3142 + 3142 + 3142, *i.e.,*
Find the value of 3142 *taken* 5 *times; i.e.,*
Find the value of 3142 × 5.

(1.) By SIMPLE ADDITION—

Having written the Multiplicand *five* times, we find by *Simple Addition* that

the SUM is 15710; but

(2.) By use of the MULTIPLICATION TABLE, the labour of performing this addition is diminished; thus, instead of saying, "2 + 2 + 2 + 2 + 2 = 10," we say, "FIVE TWOS = 10;" instead of "4+4 + 4 + 4 + 4 = 20," we say, "FIVE FOURS = 20," &c.

$$\begin{array}{r} 3142 \\ 3142 \\ 3142 \\ 3142 \\ 3142 \\ \hline \mathbf{15710} \end{array}$$

Answer, **15710.**

From which we gather that *the use of the Multiplication Table is to diminish the labour of Addition.*

(3.) By the process of SIMPLE MULTIPLICATION a further diminution of labour is effected, thus, instead of writing the multiplicand FIVE times we write it ONCE only, and indicate that it is to be made FIVE times greater by placing the digit 5 beneath its units figures. Using the Multiplication Table we proceed as follows:—

$$\begin{array}{r} 3142 \\ 5 \\ \hline \mathbf{15710} \end{array}$$

2 × 5 = 10; set down **0** and carry 1.

4 × 5 = 20, which, with the 1 carried, will make 21; set down **1** and carry 2.

1 × 5 = 5, which, with the 2 carried, will make 7; set down **7**.

3 × 5 = 15; set down **15**.

From which we gather that *the process called Simple Multiplication so uses the Multiplication Table as to find the sum of any given number taken a certain number of times much more rapidly than it could be found by Simple Addition.*

II. **Long Multiplication** *is the name given to the process generally employed when the multiplier exceeds* 12.

Ex. 2. *Find the value of* 3142 *taken* 27 *times; i.e., Find the value of* 3142 × 27.

In order to work this sum by Simple Multiplication in the same manner that Exercise 1 was worked, it is necessary to know that

$$2 \times 27 = 54,$$
$$4 \times 27 = 108,$$
$$1 \times 27 = 27,$$
$$3 \times 27 = 81;$$

but the ordinary Multiplication Table does not supply us with this information; we can, however, find it thus—

2 × 27 = 27 × 2 (§ 20, N.B.) = 54 (by Simple Multiplication); set down **4** and carry 5.

4 × 27 = 27 × 4 = 108, which, with the 5 carried, will make 113; set down **3** and carry 11.

1 × 27 = 27, which with the 11 carried, will make 38; set down **8** and carry 3.

3 × 27 = 27 × 3 = 81, which, with the 3 carried, will make 84; set down **84**.

$$\begin{array}{r} 3142 \\ 27 \\ \hline \mathbf{84834} \end{array}$$

Answer, **84834.**

Ex. 3. *Find the value of* 74362 × 457.

This example might be worked in the same manner as the last, but the process would be rather tedious and clumsy compared with the one we are now about to describe.

(1.) 74362 × 1 *unit* will give 74362 *units*:

∴ 74362 × **7** *units* will give 74362 *units* × 7, *i.e.*, (by Simple Multiplication), **520534 units.**

(2.) 74362 × 1 *ten* will give 74362 *tens*;

∴ 74362 × **5** *tens* will give 74362 *tens* × 5, *i.e.*, (by Simple Multiplication) **371810 tens.**

(3.) 74362 × 1 *hundred* will give 74362 *hundreds*;

∴ 74362 × **4** *hundreds* will give 74362 *hundreds* × 4, *i.e.*, (by Simple Multiplication), **297448 hundreds.**

Hence—

74362 × (7 *units* + 5 *tens* + 4 *hundreds*) =
520534 *units* + 371810 *tens* + 297448 *hundreds* =
520534 + 3718100 + 29744800 =
33983434, *Answer.*

Looking back over this working, we notice that we have proceeded thus—

	74362
	457
74362 × 7 units =	520534
74362 × 5 tens = 371810 tens = (§ 9, N.B.)	371810
74362 × 4 hundreds = 297448 = (§ 9, N.B.)	297448
	33983434

From which we derive the following Rule—

RULE FOR SIMPLE LONG MULTIPLICATION:—

1. Multiply the whole multiplicand by the *Units figure* **of the multiplier, as in short multiplication, and write the product beneath the multiplier,** *units under units, tens under tens, &c.*

Thus, in the above example, 74362 × 7 = 520534.

2. Multiply the whole multiplicand by the *tens figure* **of the multiplier, and write the product beneath the last product,** *taking care, however, that the units figure of the present product comes directly under the present multiplier.*

Thus, 74362 × 5 = 371810, whose units figure, 0, comes directly beneath the present multiplier, 5.

3. Multiply the whole multiplicand by the ***hundreds figure*** **of the multiplier, and write the product beneath the last product,** ***taking care also that the units figure of the present product comes directly under the present multiplier.***

Thus, 74362 × 4 = 297448, whose units figure, 8, comes directly beneath the present multiplier, 4.

4. If the multiplier contains other figures, treat them in a similar manner, and then

5. Add up the whole column of products, and their sum is the answer.

N.B.—1. The working of the following examples deserve notice—

Ex. 4. 20316 × 705.

```
   20316
     705
 -------
  101580
 142212
 -------
 14322780, Answer.
```

Ex. 5. 32407 × 6005.

```
   32407
    6005
 -------
  162035
 194442
 -------
 194604035, Answer.
```

Ex. 5. 729 × 63817 = 63817 × 729 (§ 46).

```
   63817
     729
 -------
  574353
 127634
 446719
 -------
 46522593, Answer.
```

Ex. 6. 700013 × 54836 = 54836 × 700013.

```
   54836
  700013
 -------
  164508
  54836
 383852
 -------
 38385912868, Answer.
```

Ex. 7. 24316 × 84000.

```
   24316
   84000
 -------
  97264
 194528
 -------
 2042544000, Answer.
```

Ex. 8. 319200 × 730000

```
   319200
   730000
 -------
    9576
  22344
 -------
 233016000000, Answer.
```

Ex. 9. 3147 × 17.

```
   22029
   3147
 -------
Answer, 53499
```

This is an abbreviated form of the usual method, thus—

```
   3147
     17
 -------
  22029
  3147
 -------
  53499
```

Ex. 10. 60315 × 21.

Since 21 = 7 × 3, we may proceed thus—

```
      60315
          7
     422205 = 7 times 60315.
          3
Ans., 1266615 = (3 times 7 times, i.e.,) 21 times 60315.
```

N.B.—2. It is not absolutely essential that we should multiply by the units' figure *first*; an inspection of the following methods of working an example will show that we can use any digit of the multiplier as the initial multiplier, provided only that in every case the units' figure of the product be placed exactly under the figure of the multiplier by which that product was obtained, thus—

Ex. 11. 64109 × 70638.

(1.) Beginning with the *Hundreds'* figure.

```
        64109
        70638
     384654
       512872
   448763
      192327
   4528531542, Answer.
```

(2.) Beginning with the *Ten Hundreds'* figure.

```
        64109
        70638
   448763
      192327
     384654
       512872
   4528531542, Answer.
```

Exercise 8.

Exercises in Simple Multiplication.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
3244	6381	27313	61284	91827	38241	423618	7274619
3	4	5	7	6	8	9	11

(9.)	(10.)	(11.)	(12.)	(13.)	(14.)
8134962	61642853	99886675	381406	7060080	3049273
12	12	11	9	8	11

(15.)	(16.)	(17.)	(18.)	(19.)	(20.)
60305800	23000909	46382	37981	80639	740632
12	10	26	35	68	95

(21.)	(22.)	(23.)	(24.)	(25.)	(26.)
32879604	63709	463007	2386090	582314	916378
67	682	719	80736	68243	43096

(27.)	(28.)	(29.)	(30.)
251700003	40830600	269300408	814236008
6281769	3058792	300670009	90968774

31. Find the continued product of 27, 36, and 9040.

32. Find the sum of 1084 taken 903 times.

33. Add together 7004, 90603, 91907, and add to the sum 35 times itself.

34. If a man receives a pension of 85 pence per week, how many pence will he receive in a year of 52 weeks.*

35. A mile contains 1760 yards, and 36 inches make 1 yard; how many inches are there in 51 miles?*

23. Extension of the Rule for Simple Multiplication to find the Rule for Compound Multiplication.

Ex. 1. *Multiply* £4, 3s. 1d. *by* 4.

1. *By Compound Addition* we find the Answer is £16, 12s. 4d.
2. *By use of the Multiplication Table* we proceed thus—

1d. × 4 = 4d. Set down **4**d.
3s. × 4 = 12s. Set down **12**s.
£4 × 4 = £16. Set down **£16**.
All which is most briefly put by

3. *The method of Compound Multiplication,* thus—

£	s.	d.
4	3	1
4	3	1
4	3	1
£16	**12**	**4**

£	s.	d.
£4	3	1
		4
£16	**12**	**4**

* By such sums as this the teacher should already begin to prepare his pupils for the tables of Weights and Measures.

Ex. 2. *Multiply* £16, 4s. 2¼d. *by* 7.

By Compound Multiplication we have—

	£16	4	2¼
			7
Answer,	**£113**	**9**	**3¾**

(1.) ¼d. × 7 = 7*f.* = 1¾d. Set down ¾d. and carry 1d.

(2.) 2d. × 7 = 14d., which with the 1d. carried = 15d. = 1s. 3d. Set down **3**d. and carry 1s.

(3.) 4s. × 7 = 28s., which with the 1s. carried = 29s. = £1, **9**s. Set down **9**s. and carry £1.

(4.) £6 × 7 = £42, which with the £1 carried = £43. As in Simple Multiplication, set down **3** and carry 4.

(5.) 1 × 7 = 7, which with the 4 carried = 11. Set down **11**.

The RULE for COMPOUND MULTIPLICATION OF MONEY may now be stated, thus—

Having placed the multiplier under the *pence* of the multiplicand, proceed to multiply the *farthings* in the multiplicand by the given multiplier, express the product in pence and farthings; set down the *farthings* and carry the pence. In a similar way, find the product of the *pence*, increase it by the pence carried, and express the sum in shillings and pence; set down the *pence* and carry the shillings. Next, find the product of the *shillings*, increase it by the shillings carried, and express the sum in pounds and shillings; set down the *shillings* and carry the pounds. Lastly, find the product of the *pounds*, increase it by the pounds carried, and set down the sum.

Ex. 3. *Find the sum of* £13, 6s. 2½d. *taken* 16 *times—i.e., Find the valueof* £13, 6s. 2½d. × 16.

Here 16 = 4 × 4; therefore we proceed as below—

£13,	6s.	2½d.	
		4	
£53,	4s.	10d.	= £13, 6s. 2½d. × 4
		4	
Answer, **£212,**	**19**s.	**4**d.	= £13, 6s. 2½d. × (4 × 4, *i.e.*), 16.

Ex. 4. *If the wages of a workman be* £27, 4s. 3d., *what sum of money will be required to pay* 31 *such workmen?*

The answer is to be found by multiplying £27, 4s. 3d. by 31.

Now $31 = (6 \times 5) + 1$; therefore we proceed as below—

	£27,	4s.	3d.	
			6	
	£163,	5s.	6d.	= £27, 4s. 3d. × 6
			5	
	£816,	7s.	6d.	= £27, 4s. 3d. × (6 × 5, *i.e.*), 30
	£ 27,	4s.	3d.	= £27, 4s. 3d. × 1
Answer,	£843,	11s.	9d.	= £27, 4s. 3d. × (30 + 1 =) 31.

Ex. 5. *Multiply* £7, 3s. 2½d. *by* 437.

Here $437 = (10 \times 10 \times 4) + (10 \times 3) + 7$; therefore we proceed as below—

(A.)	£7,	3s.	2½d.	
			10	
(B.)	£71,	12s.	1d.	= £7, 3s. 2½d. × 10
			10	
	£716,	0s.	10d.	= £7, 3s. 2½d. × (10 × 10, *i.e.*), 100
			4	
	£2864,	3s.	4d.	= £7, 3s. 2½d. × (100 × 4, *i.e.*), 400
	£ 214,	16s.	3d.	= £7, 3s. 2½d. × (10 × 3, *i.e.*), 30 (*i.e.*, 3 times the line B.)
	£ 50,	2s.	5½d.	= £7, 3s. 2½d. × 7 (*i.e.*, 7 times the line A.)
By Addition,	**£3129,**	**2s.**	**0½d.**,	*Answer.*

Exercise 9.

Exercises in Compound Multiplication (Money).

1. £213, 4s. 2½d. × 3, 5, 7, 9, 4.
2. £6024, 8s. 5¼d. × 7, 4, 6, 8, 10.
3. £853, 12s. 6½d. × 8, 9, 10, 11, 12.
4. £3027, 10s. 3¾d. × 2, 3, 8, 5, 7, 12.
5. £23, 6s. 4½d. × 27, 36, 35, 48, 81.
6. 7s. 4½d. × 84, 108, 121, 63, 54.
7. £1, 2s. 9¼d. × 56, 64, 49, 42, 50.
8. 13s. 2½d. × 26, 31, 47, 82, 69.
9. 3¾d. × 847, 903, 650.
10. £30, 6s. 4d. × 79, 6009, 40108.

11. If the cost of 1 pound of sugar be 4¼d., what will be the price of 112 pounds?*

12. If an ounce of gold be worth £3, 17s. 10½d., what is the value of a pound of gold, supposing that 12 ounces of gold go to the pound?

13. If an army, consisting of 6040 men, cost £2, 0s. 7½d. per man per month, what is the cost of maintaining this army for 5 months?

14. If the cost of educating a child for 1 year be £1, 11s. 1½d., what will it cost to maintain a school of 205 children for 3 years?

15. If a bookseller buys 603 novels at 2s. 1¼d. each, 410 histories at 11¾d. each, 85 magazines at 1s. 5d. each, and finds that, when he has sold them all, he has gained £65, 8s., how much money did he receive for the lot?

III. Subtraction.

24. Of Subtraction as illustrated by Addition.

(1.) If a boy has 5 marbles in one pocket and 2 in the other, we know, by Addition, that he has altogether 7 marbles; in a word, we know that

$$5 \textit{ marbles} + 2 \textit{ marbles} = 7 \textit{ marbles}.$$

Let us now suppose that this boy who has 7 marbles loses 2 of them—*i.e.*, just the number he had in one of his pockets; it is clear he will then have remaining as many marbles as he had in the other pocket—*i.e.*, 5 marbles; so that we find that

$$7 \textit{ marbles} - 2 \textit{ marbles} = 5 \textit{ marbles}.$$

In the same manner we may show that

$$7 \textit{ marbles} - 5 \textit{ marbles} = 2 \textit{ marbles};$$

and proceeding in this way we find that, *when two numbers have been added together, if we reduce their sum by one of the numbers, the other number will be left as a remainder*—

$$\begin{aligned} \textit{e.g.},\ & 63 + 26 = 89 \\ \therefore\ & 89 - 63 = 26 \\ \text{and}\ & 89 - 26 = 63. \end{aligned}$$

From this it follows that *Subtraction is a method of finding what number must be added to a given number in order to produce another given number.*

---

* The teacher should here explain that 1 *hundredweight* = 112 *pounds*.

(2.) But the above expression,

$$89 - 63 = 26,$$

may be taken to mean that when 89 has been reduced by 63, there yet will remain 26.

From which it follows that *Subtraction is a method of finding what number will remain when one given number has been taken from another given number.*

(3.) Since $89 = 63 + 26$, it is clear that 89 is *greater* than 63 *by* 26, so that Subtraction may be again described as a *method of finding by how many one number exceeds another.*

(4.) All three of these propositions may be included in this one—*Subtraction is a method of finding the difference between two given numbers.*

25. That Subtraction is the converse of Addition; leading to the Rule for Simple Subtraction.

From a consideration of § 24, we see that, whereas by Addition we find a number equal to certain other numbers taken together (*e.g.*, that $313 + 672 = 985$), by Subtraction we find what number remains when one of these numbers has been taken from their total (*e.g.*, that $985 - 313 = 672$).

We express this by saying that "*Subtraction is the converse of Addition.*"

Now, it has been already stated that $672 + 313 = 985$; this was found by Simple Addition; and we already know enough to inform us that *if* 985 *be diminished by* 672, *there will* 313 *remain.*

If the question were now put to us "*What number will remain when* 672 *has been taken from* 985?" we should reply "**313**," because we know this was the number added to 672 to produce 985. It is the province of Subtraction to supply a rule for finding this number **313 when we are totally ignorant of it.** Let us endeavour to find this rule, and in this endeavour what can aid us more than an examination of the process by which 672 and 313 were added together to produce the sum 985? Now,

$$\begin{aligned}
672 &= 6 \textit{ hundreds} + 7 \textit{ tens} + 2 \textit{ units} \\
\text{and } 313 &= 3 \textit{ hundreds} + 1 \textit{ ten} + 3 \textit{ units} \\
\text{Their } \textit{sum}, 985 &= (6 + 3) \textit{ hundreds} + (7 + 1) \textit{ tens} + (3 + 2) \textit{ units} \\
&= 9 \textit{ hundreds} + 8 \textit{ tens} + 5 \textit{ units} \\
\therefore (\S\ 24)\ 985 - 672 &= (9 - \mathbf{6}) \textit{ hundreds} + (8 - \mathbf{7}) \textit{ tens} + (5 - \mathbf{2}) \textit{ units} \\
&= 3 \textit{ hundreds} + 1 \textit{ ten} + 3 \textit{ units} \\
&= 313, \text{ where}
\end{aligned}$$

(1.) The *units'* figure, **3**, was found by taking 2, the units' figure in the subtrahend, from 5, the units' figure in the minuend.

(2.) The *tens'* figure, **1**, was found by taking 7, the tens' figure in the subtrahend, from 8, the tens' figure in the minuend.

(3.) The *hundreds'* figure, **3**, was found by taking 6, the hundreds' figure in the subtrahend, from 9, the hundreds' figure in the minuend.

Let us now take another example, more complex than this last, and see whether the same method will give us the required result.

Ex. 2. *By Simple Addition* we have—

$$\begin{aligned} 4625 + 3628 &= (4 + 3)\ \textit{thousands} + (6 + 6)\ \textit{hundreds} + (2 + 2)\ \textit{tens} \\ &\quad + (5 + 8)\ \textit{units} \\ &= 7\ \textit{thousands} + 12\ \textit{hundreds} + 4\ \textit{tens} + 13\ \textit{units} \\ &= 7\ \textit{thousands} + 1\ \textit{thousand}\ 2\ \textit{hundreds} + 4\ \textit{tens} + 1\ \textit{ten}\ 3 \\ &\quad \textit{units} \\ &= (7 + 1)\ \textit{thousands} + 2\ \textit{hundreds} + (4 + 1)\ \textit{tens} + 3\ \textit{units} \\ &= 8\ \textit{thousands} + 2\ \textit{hundreds} + 5\ \textit{tens} + 3\ \textit{units} \\ &= 8253. \end{aligned}$$

Therefore, 8253 − 4625 = 3628 (§ 24).

∴ Let the question stand thus—

"*What is the difference between* 8253 *and* 4625?

Arranging the subtrahend beneath the minuend, units under units, tens under tens, &c., we find a difficulty at the outset, for—

$$\begin{array}{r} 8253 \\ 4625 \\ \hline \mathbf{3628} \end{array}$$

(1.) *In the units' column* we are required to take 5 units from 3 units, which is impossible; but, looking above, we notice that this 3 is simply the *units'* figure of 13, the 1 *ten* of which is included in the 5 *tens*. Therefore, taking back this *ten*, we make our 3 into 13, from which we subtract the 5, and set down **8**, the remainder.

(2.) *In the tens' column* we have to take from 5 *tens* not only the 2 *tens* in the subtrahend, but also the 1 *ten* added to the 3 in the last operation; *i.e.*, from the 5 we have to take (2 + 1, *i.e.*,) 3. This we do, and set down the remainder, **2**.

(3.) *In the hundreds' column* comes another difficulty; from the 2 hundreds in the minuend we are required to take the 6 hundreds in the subtrahend. This being impossible, we go back as before to see how this 2 was produced, and we find that it is the units' figure of 12 (signifying 12 *hundreds*), the 1 *ten* (signifying *ten hundreds*), of which is included in the 8 (signifying 8 *ten hundreds*, *i.e.*, 8 *thousands*). Therefore, taking back this *ten*, we make our 2 into 12, from which we subtract the 6, and set down **6**, the remainder.

(4.) *In the thousands' column* we have to take from 8 *thousands* not only the 4 *thousands* in the subtrahend, but also the 1 *thousand* (*i.e.*, 10 *hundreds*) added to the 2 *hundreds* in the last operation; *i.e.*, from the 8 we have to take (4 + 1, *i.e.*,) 5. This we do, and set down the remainder, **3**.

Answer, **3628**, as above.

By the help of the preceding examples the pupil will be able to understand the following Rule—

RULE FOR SIMPLE SUBTRACTION:—

Having arranged the subtrahend beneath the minuend, units under units, tens under tens, &c., proceed as follows—

1. Take the *units* in the subtrahend from the units in the minuend, and place the figure which expresses the *difference* in the units' place of the answer.

2. Proceed in the same way with the *tens*, *hundreds*, &c., placing the remainder in the *tens'*, *hundreds'*, &c., places of the answer.

3. If a figure in the subtrahend exceeds the figure in the corresponding place of the minuend, take a 1 and place it on the left of the figure in the minuend. From the number thus formed, subtract the number expressed by the figure in the subtrahend; and, having found the *difference*, express it by a figure in the corresponding place of the answer, and *carry* 1.

Add this 1 to the next figure on the left in the subtrahend, and take the sum from the corresponding figure in the minuend.

This Rule may be illustrated by an example, thus—

From 73826 *take* 8375.

In the units' column—5 from 6 leaves 1; set down **1**.
In the tens' column—7 from 12 leaves 5; set down **5** and carry 1.

$$\begin{array}{r} 73826 \\ 8375 \\ \hline \mathbf{65451} \end{array}$$

In the hundreds' column—(3 + 1) from 8 = 4 from 8, which leaves 4; set down **4**.
In the thousands' column—8 from 13 leaves 5; set down **5** and carry 1.
In the ten thousands' column—The one carried being taken from 7 leaves 6; set down **6**.

∴ *Answer* = **65451**.

N.B.—The correctness of the working of sums in Subtraction may (§ 24) be proved by adding together the subtrahend and the difference, the sum of which ought to reproduce the minuend.

26. Of the two kinds of Subtraction.

Like Addition, Subtraction is of two kinds, *Simple* and *Compound*, and the same distinction obtains as in Addition. We have already found the *Rule for Simple Subtraction;* in the next paragraph we shall endeavour to extend this Rule to find the Rule for *Compound Subtraction of Money.*

Exercise 10.

Exercises in Simple Subtraction.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
78	649	876	983	631	2313	70134	120318
35	231	234	746	218	1612	6809	60907

(9.)	(10.)	(11.)	(12.)	(13.)
241016	700614	708316	360812	7760004
83091	8093	974	994	3896281

14. Find the difference between 603827 and 5092.

15. By how many does 31402003 exceed 705816?

16. How many must be taken from three hundred millions to leave a remainder of four hundred thousand and eight?

17. How many recruits are required to raise an army of twenty thousand six hundred and four to seventy-five thousand and forty?

18. An army of a hundred thousand men lost twenty-two thousand and sixty in battle, one thousand six hundred and ninety-two by an epidemic, forty-nine by desertion, and six thousand and fourteen were disabled and sent home, how many men must be sent out as reinforcements in order to make up the army to its original strength?

19. A merchant receives on Monday £3016, 14s. 7d., on Tuesday £23900, 8s. 2d., on Wednesday £601, 2s. 5d,, on Thursday £1803, 9s. 6d., on Friday £270678, 5s. 4d., and on Saturday £4500. After expending £135284 he reckons up the balance, and how much does he find it to be?

20. Find the sum of 96, 38, and 41; multiply it by the difference between 10000 and 3 *times* 681, and say how many must be taken from the product to make the difference exceed 43 *times* 612 by 9100.

27. Of Compound Subtraction as illustrated by "Compound Addition" and the "Rule for Simple Subtraction," leading to the "Rule for Compound Subtraction."

Ex. 1. *Find the sum of* £4, 2s. 6d. *and* £7, 8s. 2d.

By Compound Addition we have—

£4, 2s. 6d. + £7, 8s. 2d. = £(4 + 7) + (2 + 8) s. + (6 + 2) d.
= £11 + 10s. + 8d.
= £11, 10s. 8d., *Answer.*

∴ £11, 10s. 8d. − £4, 2s. 6d. = £7, 8s. 2d. (§ 24.)

Therefore, let the question be—

Find the difference between £11, 10s. 8d. *and* £4, 2s. 6d.

Placing the less quantity below the greater as in Simple Substraction, we find that (1.) *an application of the same process as that by which we proceeded in working sums in Simple Subtraction gives us the required answer; for—*

£	s.	d.	
11	10	8	
4	2	6	
£7	8	2,	*Answer.*

8d. − 6d. = 2d. *Set down* **2**d.
10s. − 2s. = 8s. *Set down* **8**s.
£11 − £4 = £7. *Set down* **£7**.
} *Answer*, £7, 8s. 2d.

(2.) We are confirmed in believing that such is the right method of procedure by looking back to see how the concrete numbers, 8d., 10s., and £11, were first obtained, for we notice that—

(*a.*) The **8**d. was found by adding together 6d. and 2d.
Therefore 8d. − 6d. = 2d. (§ 24.)

(*b.*) The **10**s. was found by adding together 2s. and 8s.
Therefore 10s. − 2s. = 8s. (§ 24.)

(*c.*) The **£11** was found by adding together £4 and £7.
Therefore £11 − £4 = £7 (§ 24).

From this example we gather that—

The difference between two sums of money = the difference of their PENCE *+ the difference of their* SHILLINGS *+ the difference of their* POUNDS.

Let us see how this Rule applies in another Example—

Ex. 2. *Find the sum of* £8, 13s, 10d. *and* £5, 10s. 8d.

By Compound Addition we have—

£8, 13s. 10d + £5, 10s. 8d. = £(8 + 5) + (13 + 10)s. + (10 + 8)d.
= £13 + 23s. + 18d.
= £13 + £1, 3s. + 1s. 6d.
= £(13 + 1) + (3 + 1)s. + 6d.
= £14 + 4s. + 6d.
= £14, 4s. 6d.

∴ £14, 4s. 6d. − £8, 13s. 10d. = £5, 10s. 8d. (§ 24.)

Therefore, let the question stand thus—

Find the difference between £14, 4s. 6d. *and* £8, 13s. 10d.

£	s.	d.
14	4	6
8	13	10
£5	**10**	**8**

Here we find a difficulty at starting, for—

(1.) *In the pence column* we are required to take 10d. from 6d., which is impossible. But, looking above, we notice that this 6d. is simply the *pence* of 1s. 6d., the 1s. of which is included in the 4s. Therefore, taking back this *shilling*, we make our 6d. into (1s. 6d., *i.e.*,) 18d., from which we subtract the 10d., and set down **8**d., the remainder.

(2.) *In the shillings' column* we are again in difficulty, having here to take (13s. + the 1s. added to the 6d. in the last operation, *i.e.*,) 14s. from 4s., which is impossible. But once more looking back, we notice that this 4s. is simply the *shillings* of £1, 4s., the £1 of which is included in the £14. Therefore, taking back this *pound*, we make our 4s. into (£1, 4s., *i.e.*,) 24s., from which we subtract the 14s., and set down **10**s., the remainder.

(3.) *In the pounds' column* we take from £14 (the £8 increased by the £1 added to the 4s. in the last operation, *i.e.*,) £9, and set down **£5**, the remainder.

Answer, **£5, 10s. 8d.**

N.B.—The working of this sum might be illustrated thus—

£14, 4s. 6d. = £13, 23s. 18d.
£ 8, 13s. 10d. = £ 8, 13s. 10d.

By Simple Subtraction, £5, 10s. 8d., *Answer.*

The preceding Examples will prepare the pupil for the following RULE FOR COMPOUND SUBTRACTION OF MONEY :—

Having arranged the subtrahend beneath the minuend, pence under pence, shillings under shillings, and pounds under pounds, proceed as follows—

1. Take the *pence* in the subtrahend from the pence in the minuend, and place a figure representing the difference in the pence place of the answer.

2. Proceed in the same way with the *shillings* and *pounds*, placing the remainder as shillings and pounds in the answer.

3. If the pence in the subtrahend exceed the pence in the minuend, increase the latter by *one shilling* taken from the shillings in the minuend, and, from the *sum* thus formed, take the pence in the subtrahend, express the *remainder* as pence in the answer, and carry 1.

Add this 1 to the shillings in the subtrahend, and proceed in a similar manner to find the remainder of the answer.

N.B.—1. *Farthings* are treated in a similar manner, as in the following example :—

Ex. 3. *From* 9s. 3¼d. *take* 3s. 7½d.

(1.) ¼d. − ½d.—impossible.
∴ 1¼d. − ½d. = 5f. − 2f. = 3f. = ¾d.—carry 1d.
(2.) 3d. − (7d. + 1d.) = 3d. − 8d.—impossible.
∴ 1s. 3d. − 8d. = 15d. − 8d. = **7**d.—carry 1s.
(3.) 9s. − (3s. + 1s.) = 9s. − 4s. = **5**s.

9s. 3¼d.
3s. 7½d.
Answer, **5**s. **7¾**d.

Answer, **5**s. **7¾**d.

N.B.—2. From the above rules the word "*borrow*" has been carefully excluded, as likely to engender an erroneous idea of the principle upon which the rule for Subtraction is founded. Let us illustrate—

A person possessing several bank notes for £10 each, and also £4 in gold, may wish to discharge a debt of £8. To do this he must *convert* one of his notes into cash, and from the 14 sovereigns thus obtained he can take the 8 sovereigns necessary for the discharge of his debt. Now,

it would be a gross misuse of language to say he *borrowed* the ten sovereigns; he did no such thing, he simply *converted* one of his notes into cash. The same is true in Simple Subtraction; when the units in the minuend are less than are required to enable us to take from them the units in the subtrahend, we do not in reality "*borrow*" a *ten*, we *take* one of the tens in the minuend, express it as units, and add it to the units—if any—already in the minuend.*

The same objection is fatal to the method of solving the difficulty on the principle that *when the minuend and subtrahend are each increased by* TEN *their difference is the same as that between the original minuend and subtrahend.* This proposition is, of course, true, but, in the first place, it requires much explanation and illustration to render it intelligible to children; and in the next place, it is not only *not necessary* to a complete understanding of the subject, but really tends to confuse it; besides, upon no such axiom as this is the rule for Subtraction founded.†

Such methods of "*proof of the process*" as these are ingenious, and have a legitimate use in CONFIRMING the rule; but as providing *primary* REASONS for its process we think they cannot be too strongly condemned.

N.B.—3. The method of working the following example deserves notice:—

Ex. 4. *From* £7, 3s. 4½d. *take* £2, 7s. 9¼d.

	£7, 3s. 4½d.
	£2, 7s. 9¼d.
	£4, 15s. 7¼d.

(1.) $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$d.

(2.) 4d. − 9d.—impossible; therefore take 1s.—*i.e.*, 12d., from the 3s.; now, instead of saying 1s. 4d. − 9d., say 1s. − 9 = 3d., and 3d. + 4d. in the minuend = **7**d. Here, instead of adding the *shilling* to the 4d., and then subtracting the 9d., we take the 9d. from the *shilling*, and add the 4d. to the remainder.

(3.) 3s. − (7s. + 1s., *i.e.*) 8s.—impossible; take £1, *i.e.*, 20s., from the £7; then 20s. − 8s. = 12s., and 12s + 3s. in the minuend = **15**s., &c.

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* This illustration by means of a TEN POUND bank note is perhaps as useful a one as could be found. In making clear this point to children, the author has been materially assisted by a kind of Abacus invented by Mr Earp, of Battersea.

† The following is an example of the explanation here condemned:—

	1376
	438
	938

6 − 8—impossible; add 10 to the 6; 8 from 16 leaves **8**. Here we increase the *minuend* by 10; we must therefore now increase the *subtrahend* by 10. We therefore have 7 *tens* − (3 + 1, *i.e.*) 4 *tens* = **3** *tens*.

N.B.—4. It might be pointed out that the process of *Compound* Subtraction is in reality the same as that of *Simple* Subtraction—viz., that when the figure in the minuend is exceeded by that in the subtrahend, we always get over the difficulty *by taking one unit of the next higher denomination, expressing it as units of the denomination under consideration*, and *adding the number of these units to that of the units in the minuend*, and then proceeding to *the actual process of subtraction;* bearing in mind, however, this one distinctive difference that whereas in **Simple Subtraction** a unit of any denomination always contains TEN units of the next lower denomination—*e.g.*, 1 *thousand* = **10** *hundreds;* 1 *hundred* = **10** *tens*, &c., in **Compound Subtraction** the number of units so contained is different with different units—*e.g.*, 1 *pound* = **20** *shillings;* 1 *shilling* = **12** *pence*, &c.

The same observation, of course, holds good with reference to Addition, Multiplication, and Division, from which it follows that, if the same law held good with units in Compound Rules as obtains in Simple Rules, the processes of Addition, Subtraction, &c., would be the same in Compound as in Simple Rules.

Exercise 11.

Exercises in Compound Subtraction of Money.

(1.)			(2.)			(3.)			(4.)			(5.)		
£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
84	17	9	79	10	8	4020	6	10½	7031	10	6	30019	2	4
36	5	7	38	2	7	208	3	4¼	2038	17	4	681	19	6

(6.)			(7.)			(8.)			(9.)		
£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
3610	0	5	501003	3	2¼	40010	13	6	2010006	3	4½
861	9	1¼	6018	1	10½	801	7	9¾	999	3	8¾

(10.)			(11.)			(12.)			(13.)		
£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
100000	2	3½	443106	2	0	2040	3	0	3000	0	0
814	19	7	8409	17	0¼		12	6½	20	18	0¼

(14.)			(15.)			(16.)			(17.)		(18.)	
£	s.	d.	£	s.	d.	£	s.	d.	s.	d.	s.	d.
17000	0	9	20	0	0	201	0	0¾	16	0	17	2¼
16089	19	9¾	3	0	0¼	80	7	1		8½	7	2¾

19. Find the difference between £10 and £1, 17s. 2¾d.

20. What sum remains when £300, 10s. has been taken from £700, 0s. 1d.?

21. By how much does £80 exceed £14, 3s. 4½d.?

22. How much must be added to £700, 14s. 6d. to make £1300, 2s.

23. A house worth £300 is exchanged for a house and furniture. If this latter house be valued at £230, 19s., what was the value attached to the furniture?

24. If a horse and cart be worth £83, the cart itself being valued at £5, 16s. 8d., what is the value of the horse?

25. A horse, a cow, and a sum of money being exchanged for a piece of land worth £93, 10s., what was that sum of money if the horse be worth £40, 7s., and the cow £19, 10s. 6d.?

26. If the receipts at a museum be £40, 7s. 6d. in January, £25, 9s. 3d. in February, and £51, 0s. 7d. in March, how much must be taken in April to make a total of £180, 7s. 7d. in the four months?

27. A gentleman deposited £800, 10s. 6d. in the bank, and, when he withdrew it, received £867, 0s. 5d., how much of this was interest?

28. If the sum of £60, 3s. 9½d. be increased to 19 times its present value, and then be diminished by £400, how much must be added to 29 times 7s. 3¾d. to make it equal the remainder?

IV. Division.

28. Of Division as illustrated by Multiplication and Subtraction.

1. If a boy has 5 tops, each worth 2d., we find by Simple Multiplication that the total value of the 5 tops is 10d.; we, in fact, know that—

$$2\text{d.} \times 5 = 10\text{d.}$$

Let it now be supposed that the following question is put: *If a boy had* 10d., *how many tops could he buy at* 2d. *each?*

(*a.*) *By Simple Multiplication* we find that 2d. must be multiplied by 5 to make 10d., therefore he can buy 5 tops.

Answer, **5**.

To find the answer as above, we are only required to know *how many* TWOS *make* TEN, and the Multiplication Table tells us that FIVE *twos* make *ten*.

But the answer can also be found—

(*b.*) *By Simple Subtraction*, thus: It is clear that for every top purchased the boy must take 2d. from his 10d. Let us, therefore, see how many *twopences* can be taken from *tenpence*, and we shall then know how many tops can be purchased—*i.e.*, we shall know the answer.

We find on trial that 5 twopences can be taken from 10d. Therefore the answer is **5**.

We here have the processes of Multiplication and Subtraction illustrating each other, and they will also assist us in understanding the Rule called **Division**, as follows—

10d.
2d.—1
8d.
2d —2
6d.
2d.—3
4d.
2d.—4
2d.
2d.—5

2. It is evident that the concrete number 10d. may be considered as the *sum* of the FIVE *equal* concrete numbers, 2d., 2d., 2d., 2d., 2d., and that it may be divided into such five numbers. If we wish to find how many such numbers were contained in the *sum* tenpence, we should do this by *dividing the* 10d. (*i.e.*, the total number of pence,) *by* 2 (the number of pence in each part), the expression for which is—

Divide 10 *by* 2.

or, in symbols,

Find the value of $10 \div 2$,

or, *Find the value of* $\frac{10}{2}$;

which, put into other words, is—

Find how many twos make ten.

This we can only do by means either of Subtraction, which is a very tedious process of finding the required answer, or by the use of the Multiplication Table.

From which we gather—

(1.) *That Division is a rapid method of finding results which could be obtained, though in a more tedious manner, by Subtraction.*

(2.) *That by Division we can find "How many times one number contains, or is contained in, another;" "How many times one number may therefore be taken from another," &c. &c.*

(3.) *That Division, like Multiplication, requires a knowledge of the Multiplication Table; is, in fact, a Rule based upon the information contained in that table.*

(4.) *That Division is the converse of Multiplication, just in the same way that Subtraction is the converse of Addition.*

29. Of the Rule for Simple Division, as deduced from Examples.

Ex. 1. $243 \times 2 = 486$.

Therefore $486 \div 2 = 243$.

∴ Let the question stand thus—

How many times does 486 *contain* 2; *i.e.*,

Find the value of $486 \div 2$.

We know already that the answer is 243, but it is the province of Division to supply a rule for finding this number, 243, when we are totally ignorant of it.

Arranging the Divisor and Dividend thus—

$$\begin{array}{r|l} 2 & 486 \\ \hline & 243 \end{array}$$

we observe that—

(1.) *Since the* 4 *hundreds* was obtained by multiplying 2 *hundreds* by 2, as above, the **2** *hundreds* may be reproduced by *finding how many* TWOS *are contained in* FOUR—*i.e.*, by *dividing* 4 *by* 2. The Multiplication Table gives us 2 as the quotient, we therefore place **2** in the hundreds' place of the answer.

(2.) Again, *since the* 8 *tens* was obtained by multiplying 4 *tens* by 2, as above, the **4** *tens* may be reproduced by *finding how many* TWOS *are contained in* EIGHT—*i.e.*, by *dividing* 8 *by* 2. The Multiplication Table gives us 4 as the quotient, we therefore place **4** in the tens' place of the answer.

(3.) Lastly, *since the* 6 *units* was obtained by multiplying 3 *units* by 2, as above, the **3** *units* may be reproduced by *finding how many* TWOS *are contained in* SIX—*i.e.*, by *dividing* 6 *by* 2. The Multiplication Table gives us 3 as the quotient, we therefore place **3** in the units' place of the answer

Answer, **243**.

Here we have found the answer by finding how many times the divisor is contained in succession by the hundreds, tens, and units of the dividend.

Let us now take another example, and see whether the same method of procedure will produce the required quotient.

Ex. 2. $456 \times 3 = 1368$

Therefore $1368 \div 3 = 456$.

∴ Let the question stand thus—

Find the value of 1368 ÷ 3.

Proceeding as before, we meet with a difficulty at starting, for *in the thousands' column* we have to divide 1 by 3, which is impossible. But, looking back to see how this figure 1 was obtained, we notice that it is simply the 1 *ten* of the number 13, signifying 13 *hundreds.* We therefore once more express the 1 *thousand* 3 *hundreds* as 13 *hundreds*, and proceed to divide this 13 by 3. We find that 13 contains 4 *threes* and 1 over: we therefore *set down* **4** in the hundreds' place of the answer. But what is to be done with the 1 *over?*

3 | 1368
Answer, 456

Looking back to see how it was produced, we find that it is the 1 *ten* of the number 16, signifying 16 *tens.* We therefore once more express the 1 *hundred* 6 *tens* as 16 *tens*, and dividing this 16 by 3, find it contains 5 *threes* and 1 over; we therefore *set down* **5** in the tens' place of the answer, and carry 1 as in the last operation.

This 1 we find to be the 1 *ten* of the number 18, signifying 18 *units.* Therefore, dividing 18 by 3, we put down the quotient **6** in the units' place of the answer. *Answer,* **456.**

Ex. 3. 487 × 6 = 2922 ∴ 2922 ÷ 6 = 487.

Therefore let the question stand thus—

How many times is 6 *contained in* 2922?

We *know* the answer to be 487, but our present purpose is to find this answer, supposing ourselves to be totally ignorant of it.

6 | 2922
487

(1.) Seeing that 2, the *thousands'* figure of the dividend, is less than 6, the divisor, we conclude at once that *the answer is less than one thousand,* and will therefore be *expressed by less than four figures,* because any whole number of four figures must be at least a thousand.

We therefore conclude that *when the divisor is greater than the first figure on the left of the dividend, the first figure on the left of the answer should be placed under the second* (*sometimes* **the third**) *figure of the dividend, counting from the left.*

The difficulty is now overcome by reducing the thousands to hundreds, thus—

2 thousand 9 hundred = 29 hundreds (§ 9, N.B.), and, since 29 ÷ 6 = 4, and 5 over,

29 hundreds ÷ 6 = 4 hundreds, and 5 hundreds over, therefore put 4 under the 9 in the dividend—*i.e.*, put **4** in the *hundreds'* place of the answer.

Or, which comes to the same thing, proceed by the following Rule—

RULE.—When the first figure on the left of the dividend is less than the divisor, take the first two (or three, if necessary,) figures of the dividend, find how many times the number expressed by these figures contains the divisor, place beneath the second (or third, if three be taken,) figure from the left of the dividend a figure to express the times the divisor is contained in the number, and carry the figure which is over.

The remainder of the answer may now be found by a process which depends upon the foregoing explanations; and, to put the whole process clearly, we shall now work the example completely through. Thus—

6 | 2922
Answer, **487**

(1.) *Sixes in two? i.e., How many sixes are there in two?* None.

∴ Leave the space beneath the 2 vacant.

(2.) *Sixes in twenty-nine? Four, and five over.*

∴ Put **4** beneath the 9, and carry 5.

(3.) Bringing down 2, the next figure in the dividend, and placing it on the right of the 5 carried, we get 52.

Sixes in fifty-two? Eight, and four over.

∴ Put **8** beneath the 2 *tens*, and carry 4.

(4.) Bringing down 2, the remaining figure in the dividend, and placing it on the right of the 4 carried, we get 42.

Sixes in forty-two? Seven.

∴ Put **7** in the units' place of the answer.

Answer, **487**.

Exercise 12.

Exercises in Simple Short Division.

1. 936 ÷ 3	11. 23562 ÷ 7	21. 1364972 ÷ 9
2. 3682 ÷ 2	12. 364180 ÷ 10	22. 24316281 ÷ 7
3. 9456 ÷ 3	13. 8591768 ÷ 9	23. 90162734 ÷ 12
4. 78645 ÷ 3	14. 830690 ÷ 7	24. 827153035 ÷ 12
5. 72468 ÷ 4	15. 25796400 ÷ 8	25. 10063915617 ÷ 11
6. 89232 ÷ 4	16. 3237630 ÷ 9	26. 210863579 ÷ 12
7. 75355 ÷ 5	17. 42861903 ÷ 11	27. 8714609317 ÷ 9
8. 826408 ÷ 8	18. 2503063 ÷ 4	28. 34484076 ÷ 11
9. 272632 ÷ 8	19. 70360091 ÷ 4	29. 200638205 ÷ 12
10. 513276 ÷ 8	20. 6180123 ÷ 5	30. 815663062 ÷ 12

31. How many times does 43824 contain 4?

32. How many times may eleven be taken from three hundred and thirty-two thousand eight hundred and ninety-three?

33. If a person has 3672 farthings, to how many children could he give four farthings each?

34. If a man earns 11d. an hour, how many hours must he work to earn 385d.?

35. If Seville oranges are worth $1\frac{3}{4}$d. each, how many could be purchased for 21s.?

30. Of the several kinds of Division.

Like Multiplication, Division is of two kinds, viz.—

(1.) **Simple Division,** which is subdivided into *Simple Short Division* and *Simple Long Division.*

(2.) **Compound Division,** which also is subdivided into *Compound Short Division* and *Compound Long Division.*

In *Simple Short Division*, and in *Compound Short Division*, the divisor usually does not exceed 12; in *Long Division*, the divisor always exceeds 12.

31. Of the Rule for Compound Short Division as deduced from that for Simple Short Division.

In § 29 we have shown how to work exercises by Simple Short Division, let us now proceed to find a rule for working exercises in Compound Short Division.

In § 27, *N.B.* 4, it has been shown that the difference between the method of *Simple* Subtraction and that of *Compound* Subtraction is caused by the fact that the *former* deals always with units, of which any one is uniformly ten times greater than one of the denomination next below it, while in the *latter* no such uniform value attaches to the units dealt with.

The difference between the process of *Simple* and *Compound* Division is occasioned in the same way, as will appear from the following example—

Ex. 1. £339, 13s. $10\frac{1}{2}$d. × 6 = £2038, 3s. 3d.

∴ £2038, 3s. 3d. ÷ 6 = £339, 13s. $10\frac{1}{2}$d. Therefore let the question be, *Divide* £2038, 3s. 3d. *by* 6.

Here we have—

(1.) $20 \div 6 = 3$, and 2 *over*.

We set down **3**, but what is done with the 2 *over?*

	£	s.	d.
6 |	2038	3	3
	£339	**13**	**10½**

Proceeding by the rule, we bring down 3, the next figure in the dividend, set it beside this 2 over, and thus get 23. But what does this 23 signify? Evidently 23 *tens*, from which it is clear that the **2** *hundreds* over have been converted into **20** *tens*, and to it have been added all the *tens* in the dividend. In a word, the number remaining (*i.e.*, "over") has been converted into units of the next lower denomination, has been then increased by the units of that same denomination in the dividend, and has thus produced a new number (*i.e.*, 23) to be divided by the given divisor, 6.

(2.) $23 \div 6 = 3$, and 5 *over*.

The "5 *over*" is treated in a manner similar to that in which the "2 *over*" was treated above.

(2.) $58 \div 6 = 9$, and 4 *over*.

Set down 9, but what is to be done with the "4 *over?*" If we bring down beside it the 3, the next figure in the dividend, as we have done in the preceding operations of this example, we shall have—

43s. ÷ 6 = 7s., and 1s. over,

whereas the answer requires **13s.** (instead of 7s.), as we already know.

At this point, then, the usual method of Simple Division fails to give a correct answer to a question in Compound Division, and here is the reason—

In (1) above we saw that the "2 *hundreds over*" were converted into *tens* (*i.e.*, into *units of the next lower denomination*) by writing the 2 as the *tens'* figure of a new number, 23.

But the same result is not obtained by the same method in the case of the "4 *over*," for this 4 is £4, and is not converted into *shillings* (*i.e.*, into *units of the next lower denomination*) by making it the *tens'* figure of the new number, 43s., as we have attempted to do above.

What we want to do then is to convert £4, 3s. into *shillings*, and the number of shillings so found is **83**. Now—

83s. ÷ 6 = 13s., and 5s. over.

Therefore set down **13s.** and carry 5s.

(4.) Similarly, 5s. 3d. must be reduced to *pence* (§ 27), *i.e.*, to units *of the next lower denomination to shillings.*

The 63 pence, so found, is divided by 6, thus—

63d. ÷ 6 = 10d., and 3d. *over*.

Set down **10**d. and carry 3d.

(5.) In the same way this 3d. is reduced to 12 farthings (§ 27); then 12 f. ÷ 6 = 2 f. = ½d.

Set down ½d.

Answer, **£339, 13s. 10½d.**

The reason of the following rule is now easily seen—

RULE FOR COMPOUND DIVISION OF MONEY:—

Having arranged the divisor and the dividend in the same manner as for Simple Division, proceed as follows—

(1.) Divide the *pounds*, if any, by the divisor, as in Simple Division.

(2.) If any pounds remain, convert them into *shillings*, add the number of shillings in the dividend, and divide the sum after the manner of Simple Division.

(3.) Proceed in the same way with *pence* and *farthings*.

N.B.—The correctness of results obtained by Division can be proved by Multiplication, for *the quotient* × *the divisor* = *the dividend*, when the operation has been correctly performed.

Ex. 2. *Divide* £187, 5s. 1½d. *by* 6, *and prove the result.*

Here we have—

(1.) £187 ÷ 6 = £31, and £1 over.
Set down **£31.**

(2.) £1 and 5s. = 25s. ∴ 25s. ÷ 4 = 4s., and 1s. over.
Set down **4**s.

(3.) 1s. and 1d. = 13d.; 13d. ÷ 6 = 2d, and 1d. over.
Set down **2**d.

(4.) 1d. and ½d. = 6 f.; ∴ 6 f. ÷ 6 = 1 f.
Set down ¼d.

	£	s.	d.
6	187	5	1½
	£31	**4**	**2¼**

Answer, **£31, 4s. 2¼d.**

PROOF:

By multiplying the *quotient* by the *divisor*, we obtain the original *dividend*.

£31, 4s. 2¼d.	
6	
£187, 5s. 1½d.	

Exercise 13.

Exercises in Compound Short Division of Money.

1. 8s. 2½d. ÷ 2
2. 5s. 9d. ÷ 2
3. 6s. 7½d. ÷ 3
4. 19s. 4½d. ÷ 3
5. 17s. 8d. ÷ 4
6. 5s. 1d. ÷ 4
7. £1, 16s. 2d. ÷ 2
8. £32, 7s. 1d. ÷ 5
9. £97, 3s. 1½d. ÷ 5
10. £318, 2s. 1½d. ÷ 6
11. £1503, 10s. 7½d. ÷ 6
12. £8034, 13s. 2d ÷ 7
13. £10142, 2s. 9d. ÷ 10
14. £623, 17s. 2¼d. ÷ 10
15. £4128, 3s. 0¾d. ÷ 11
16. £63019, 2s. 1d. ÷ 7
17. £106312, 0s. 4½d. ÷ 8
18. £200601, 3s. 0d. ÷ 9
19. £13602, 0s. 0d. ÷ 12
20. £317285, 6s. 1½d. ÷ 7
21. 403106, 12s. 0½d ÷ 11

22. What sum of money multiplied by 7 will give £141, 1s. 10½d.?

23. A person bought 7 houses, and found that altogether they cost him £2107, 9s. 5¾d.; what was the cost of 1, and what would have been the cost of 13?

24. A house is sold for £287, 2s. 2¾d., and the money is divided among 11 persons, what is the share of each?

25. A cattle dealer receives £1803, 18s. for a dozen cows, what was the price of each?

26. A hospital is to be built for £4500. When £4418, 9s. has been raised, three gentlemen agree not only to find the sum still wanting, but also to provide between them a further sum of £1000 to start the institution comfortably. How much will each of these gentlemen have to subscribe?

32. Examination of the process in Short Division to find the Rule for Long Division.

Long Division is the name generally given to the process of division when the divisor exceeds 12.

It might have been observed that Long Multiplication is not a process founded upon any principle which differs from that of Short Multiplication; in fact, the latter rule might be regarded as a *particular case* of the more general rule described as "*the Rule for Long Multiplication*," which last might perhaps be better termed "THE *Rule for Multiplication*."

In the same way it will be noticed that the method of Short Division is essentially the same as that of Long Division which we are about to describe. For this reason the cases of *Simple* and *Compound* Long Division will be treated of together, as follows—

We will first take an example of Short Division, examine the several steps in its working, and by this means prepare the way for a statement of the method of Long Division.

Ex. 1. *Divide* 394891 *by* 7.

(*Fig.* 1.)

```
7 | 394891
    56413, Answer.
```

Proceeding by Short Division we have—

(1.) $3 \div 7$.

(2.) $39 \div 7 = 5$, and 4 over.

Set down **5** and carry 4.

(*Fig.* 2.)

```
7) 394891 (56413, Answer.
   35
   44
   42
    28
    28
     9
     7
     21
     21
```

Now we obtained this result thus—

a. We multiplied 7 by 5, thus obtaining 35.

b. We subtracted this 35 from 39 and found the remainder 4.

c. We set the five beneath the 9 and carried 4. But there was no particular reason why the five should be placed under the 9; so long as it was placed somewhere in such a manner as to signify 5 *tens of thousands*, our purpose would have been fully served. Let us, therefore, as in fig. 2, *place it on the right of the dividend.* We have then space beneath the 39 in which to place the 35, subtract it from 39, and put down the remainder, 4. (See fig. 2.)

(3.) $44 \div 7 = 6$, and 2 over.

Set down **6** and carry 2.

a. In fig. 1 the 6 is placed on the right of the 5 found in the last operation. *The same is done in fig.* 2.

b. In fig. 1 the 44 was obtained by bringing down the next figure in the dividend, 4, and placing it on the right of the last remainder. *The same is done in fig.* 2.

c. In fig. 1 the 7 is multiplied by 6, and the product, 42, is taken from the 44. *The same is done in fig.* 2.

(4.) $28 \div 7 = 4$. Set down **4**.

Here again the method pursued in fig. 2 is exactly the same as that pursued in fig. 1.

(5.) $9 \div 7 = 1$, and 2 over. Set down **1** and carry 2.

(6.) $21 \div 7 = 3$. Set down **3**.

From an examination of this example we see—

1. That the method of *Long* Division, as exemplified in fig. 2, is really the same as that of *Short* Division, as seen in fig. 1; the only difference being that *in the former case all the working of the process is shown, while in the latter case the results only are shown.*

2. The reason of the method employed in the following Example—

Ex. 2. *Divide* 196365 *by* 53.

```
Divisor. Dividend. Quotient.
  53) 196365 (3705, Answer.
      159
       373
       371
         265
         265
```

Placing the divisor on the left of the dividend, and leaving room for the quotient on the right of the dividend, we proceed thus—

(1.) *Fifty-threes in one?* None. Therefore—

(2.) *Fifty-threes in Nineteen?* None. Therefore—

(3.) *Fifty-threes in one hundred and ninety-six?*

We have here no table to guide us as in Example 1, so all we can do is to try such a figure in the quotient as seems likely to give us a proper number for subtracting from 196. We shall find **3** a suitable number in this case; because, when the divisor, 53, is multiplied by it, a product, 159, is produced which—

a. Does not exceed the number 196;

b. Does not, when subtracted from 196, leave a difference equal to the divisor, 53, nor one greater than 53.

If the product, 159, *had been* greater than the number 196, the trial figure, 3, would have been too great, and must have been replaced by one of less value.

If the product, 159, *had been* less than 196 by a number equal to or greater than the divisor, 53, the trial figure, 3, would have been too small, and must have been replaced by one of greater value.

Therefore, subtracting 159 from 196, we find the difference 37.

(4.) Bringing down 3, the next figure in the dividend, we get 373, therefore—

Fifty-threes in three hundred and seventy-three? Taking **7** as a trial figure, we multiply the divisor by it, and thus obtaining 371, find it fulfils the conditions mentioned above; therefore, 7 is placed in the quotient, and 371 is subtracted from 373, leaving 2 as a remainder.

(5.) Bringing down 6, the next figure in the dividend, we get 26; therefore—

Fifty-threes in twenty-six? None. Therefore put **0** in the quotient,

and bring down 5, the remaining figure in the dividend, we thus get 265; therefore—

Fifty-threes in two hundred and sixty-five? Taking 5 as a trial figure, it proves to be the right one; we therefore put **5** in the quotient.

Answer, **3705**.

Let us now take an example in Compound Long Division—

Ex. 3. *Divide* £3709, 11s. 10½d. *by* 9.

The method of Short Division (fig. 1) agrees with that of Long Division (fig. 2), step by step, as follows—

(*Fig.* 1.)

9 | £3709, 11s. 10½d.

£412, **3**s. **6**½d., *Answer.*

(*Fig.* 2.)

9) £3709, 11s. 10½d. (**£412**, **3**s. **6**½d.

36

10

9

19

18

£1

20

(20s. + 11s. =) 31s.

27s.

4s.

12

(4 × 12)d. + 10d. = 58d.

54d.

4d.

4

(4 × 4)f. + 2f. = 18f.

18f.

(1.) The £3709 is divided by 9 in fig. 2, as in the last example, and gives a quotient of £412, and £1 over.

(2.) This £1 *over* is converted into 20s., to this 20s. we added the 11s. in the dividend, thus producing 31s., and the process of Simple Division is repeated, the quotient **3** being set down as 3 *shillings*, because the dividend producing it was *shillings*.

(3.) The 4s. *over* is converted into 48d., to this 48d., are added the 10d. in the dividend, thus producing 58d., and the process of Simple Division is repeated, the quotient **6** being set down as 6 *pence*, because the dividend producing it was *pence*.

(4.) The 4d. *over* is converted into 16 f., to this 16 f. are added the 2 f. in the dividend, thus producing 18 f., and the process of Simple Division is repeated, the quotient **2** being set down as 2 *farthings*, *i.e.*, ½d., because the dividend producing it was *farthings*.

Answer, **£412**, **3**s. **6**½d.

Ex. 4. *Divide* £116396, 12s. 1¼d. *by* 29.

```
                29) £116396, 12s. 1¼d. (£4013, 13s. 6¼d.
                    116                          Answer.
                     39
                     29
                     106
                      87
                     £19
                      20
(19 × 20)s. + 12s. = 392
                      29
                      102
                       87
                       15s.
                       12
(15 × 12)d. + 1d. = 181d.
                    174d.
                      7d.
                      4
(7 × 4)f. + ¼d. = 29f.
                    29f.
```

The pounds are dealt with as in Simple Division.

The 19 *pounds over* are converted into 380 shillings, which, with the 12s. in the dividend, makes 392s.

This 392s. is divided by 29 as in Simple Division, thus—

```
29) 392s. (13s.
    29
    102
     87
     15s.
```

The 15 *shillings over* are similarly converted to 180 pence, which, with the 1d. in the dividend, make 181d.

This 181d. is divided by 29 as in Simple Division, thus—

```
29) 181d. (6d.
    174d.
      7d.
```

The 7 *pence over* are converted into 28 farthings, which, with the 1f. in the dividend, make 29f. This 29f. is divided by 29, and the quotient, 1f.—*i.e.*, ¼d., is put after the pence in the answer.

Answer, **£4013, 13s. 6¼d.**

N.B.—The method of working the following examples deserves notice—

Ex. 5. *Divide* £856, 3s. 7½d. *by* 27.

```
9 | £856,  3s. 7½d.
3 | £95,   2s. 7½d.
Answer, £31, 14s. 2½d.
```

Since 27 = 9 × 3, we
1st, Divide by 9, and
2nd, Divide the quotient, so found, by 3.

The result may be proved thus—

£31, 14s. 2½d.
3
£31, 14s. 2½d. × 3 = £95, 2s. 7½d.
9
£95, 2s. 7½d. × 9 = **£856, 3s. 7½d.** = 9 times 3 times £31, 14s. 2½d.

Ex. 6. *How many times may* 9s. 3½d. *be taken from* £1, 7s. 10½d.

1. By subtraction, we find that 3 times is the answer.

£1, 7s. 10½d.
9s. 3½d.—1
18s. 7d.
9s. 3½d.—2
9s. 3½d.
9s. 3½d.—3

2. Since 9s. 3½d. = 223 *half-pence*, and £1, 7s. 10½d. = 669 *half-pence*, the question may be rendered thus—

How many times is 223 *contained in* 669?

∴ By division, we find that 3 times is the answer.

223) 669 (**3**, *Answer.*
669

The second method is, of course, to be preferred, because, as a rule, it is the more expeditious of the two.

Exercise 14.

Exercises in Simple Long Division.

1. 738425 ÷ 25
2. 903215 ÷ 47
3. 2135589 ÷ 29
4. 642165 ÷ 31
5. 4889024 ÷ 56
6. 176429652 ÷ 42
7. 69620950 ÷ 85
8. 550801190 ÷ 17
9. 921614841 ÷ 93
10. 321060138 ÷ 29
11. 7053369600 ÷ 86
12. 901358 ÷ 216
13. 6765849 ÷ 729
14. 2826231 ÷ 537
15. 307580082 ÷ 439
16. 365243945 ÷ 605
17. 11223670536 ÷ 4008
18. 544833650 ÷ 9073
19. 5100942407 ÷ 8027
20. 60317924 ÷ 3607
21. 283601934 ÷ 9817
22. 6482791060 ÷ 24163
23. 6309506 ÷ 4173
24. 74424339 ÷ 9281
25. 367409930 ÷ 5263
26. 4288523190 ÷ 14007
27. 266508630000 ÷ 27000
28. 16345009500 ÷ 12010
29. 12865569570 ÷ 61234
30. 11742420000 ÷ 170180
31. 3182064053 ÷ 71638
32. 9072109381 ÷ 600900
33. 7703451963 ÷ 2018039

34. How many times is 67 contained in 54605?

35. If a man earns $9\frac{3}{4}$d. per hour, how many hours will it take him to earn 15795 farthings?

36. If a clock ticks two thousand five hundred and twenty times in an hour, how long will it be ticking 183960 times?

37. How many times did the wheel of a velocipede turn in a minute if in one hour it turned two thousand two hundred and twenty times?

38. How many gross are there in 3852 score? *

39. A man bought a number of articles, and sold them so as to lose $5\frac{3}{4}$d. on each; when they were all sold he found he was 1955 farthings out of pocket. How many did he buy?

40. Another man, who sold his so as to *gain* $4\frac{1}{4}$d. on each, found that, when he had sold all he bought, he had gained in all just as much as the other lost. How many did *he* buy?

41. If 47 nuts are worth 1d., what should be the price of nine thousand seven hundred and twenty-nine nuts?

42. How many times may 78 be taken from the continued product of 13, 71, and 102?

43. How many times is the product of 75 and 109 contained in the sum of 2063014, 17005000, 469, and 30214707?

44. When fifty-three has been subtracted seven hundred and six times from a certain number, it is found that eight hundred and nine still remains. What is that number, and how many times could 5461 be taken from it?

45. If the wheel of a cart be 107 inches in *circumference*,† how many times would it turn in travelling 1391 miles. [*N.B.*—1 mile=1760 yards, and 1 yard = 36 inches.]

46. A tradesman purchased a number of articles at £5 each, and sold them for quarter as much again as he gave for them. Seeing that in all he gained 525 shillings, how many did he buy?

---

* This example furnishes an opportunity for (1) teaching the meaning of "*gross*" and "*score;*" (2) preparing the pupils for an understanding of elementary *Proportion*.

† Cf. *Circumnavigate, circumvent, circumstance*, &c.; also mention incidentally the use of the words *radius, diameter*, &c., and illustrate thus—

Exercise 15.

Exercises in Compound Long Division of Money.

1. £311, 13s. 1d. ÷ 13
2. £9367, 2s. 8d. ÷ 23
3. £216404, 8s. 0d. ÷ 72
4. £504, 3s. 1¼d. ÷ 69
5. £72998, 5s. 9d. ÷ 28
6. £189056, 7s. 3d. ÷ 54
7. £21728, 15s. 8d. ÷ 604
8. £21719, 19s. 5¼d. ÷ 301
9. £9818, 7s. 6d. ÷ 980
10. £151, 14s. 1¾d. ÷ 659
11. £5655, 1s. 6¾d. ÷ 7005
12. £85072, 5s. 0d. ÷ 4080
13. £238151, 17s. 6d. ÷ 3900
14. £68191, 16s. 9½d. ÷ 4003
15. £6819263, 2s. 1¼d. ÷ 3407
16. £6307, 17s. 3½d. ÷ 7025
17. £387828 ÷ 3840
18. £1060079, 0s. 3d. ÷ 60153
19. £723586, 0s. 11d. ÷ 18076
20. £50000, 10s. ÷ 70180
21. £27086406, 7s. 1d. ÷ 90100
22. £2327606, 0s. 2d. ÷ 9347
23. £100614, 7s. ÷ 407
24. £938160, 3s. 0¼d. ÷ 72013
25. £16, 3s. 7¼d. ÷ 21
26. £1063, 19s. 2¾d. ÷ 800
27. £713082, 3s. 6¼d. ÷ 7000
28. £240391, 10s. ÷ 6300
29. £70017, 17s. 4¼d. ÷ 460
30. £3801906, 0s. 10d. ÷ 6007

31. How many times may 6¾d. be taken from £55, 7s.?

32. How many times does £179, 11s. contain 4s. 9d.?

33. How many pounds of sugar, at 5¾d. per pound, are worth 690 pounds of tea, at 3s. 2d. per pound?

34. If the total cost of one hundred and sixty-five miles of railway be £214787, 7s. 6d., what was the average cost per mile?

35. If a man saved 4s. 6¾d. per day, how much did he save in a year of 365 days? and how many years would it take him to save £832, 13s. 1½d.?

36. What is the weekly cost of a house when the annual rent is £150, and the rates come to £12, 16s. 6d.?

37. A farmer goes to market with 43 pigs and 67 sheep. He sells the former at £5, 10s. 6d. each, and the latter at such a price that the total receipts are eleven shillings greater than twice the money received for the pigs. What was the price of each sheep?

CHAPTER III.

COMPOUND RULES—WEIGHTS AND MEASURES.

I. THE ENGLISH TABLES OF WEIGHTS AND MEASURES.

33. Of the Tables of Length, Surface, and Solidity, in their connection with the Table of Time.

It has been already shown how abstract ideas of number were formed, and how the relative values of numbers came to be known and understood. In the same way it may be considered that a nation just emerging from a barbarous state would begin to feel the necessity of words to enable them to describe the length of their journeys, the width of a neighbouring river, the extent of a distant lake or forest, or the dimensions of a newly discovered cavern. In a word, they would need language to express SPACE in its three forms of

(1.) *Length—i.e.*, extent in *one* direction.

(2.) *Surface*, or *superficial area—i.e.*, extent in *two* directions—viz., length and breadth.

(3.) *Solidity*, or *cubic content—i.e.*, extent in *three* directions—viz., length, breadth, and thickness.

And, we may reasonably suppose, that they would require the language of "**Length**" first, and that the need of the language of "**Surface**" would only be felt after some considerable time. For a semi-barbarian would sufficiently express his idea of the size of any piece of land or water by saying it was *so long* and *so broad*. The same is true of the language of "**Solidity**." Thus it comes about that the names of the units in the Tables of *Surface* and *Solidity*, given below, are some of them so much like those in the Table of *Length—e.g.*, we shall have to speak of *inches* of Length, of *square inches* of Surface, and of *cubic inches* of Solidity.

There is some inconvenience in having these names so much alike; and in these days of greater civilisation people have learned to complain about it, and have devised plans for remedying this and other inconveniences in our Tables of Weights and Measures.

There is a new system used in France which avoids these difficulties, and which many people are desirous of introducing into England. (See "Advanced Arithmetic," § 38.)

34. Of the English Table of Time.

But long before the need of words to express SPACE is felt, man has need of language to express TIME; and so we often find savage nations expressing *distance* by means of their language of time—*e.g.*, that a friendly tribe lives so many *suns* away—*i.e.*, distant so many *days'* journey. Moreover, we often hear in our own country that certain Alpine spots are so many *hours* apart. For this reason we shall give first

The English Table of Time.

Now, if we desired to express how long ago a friend died, or how long one had lived, we should express the time in *years*. In this case we should use *a year* as the unit by which *to measure* the time under consideration. And we shall find that *in each of the Tables of Weights and Measures there is one principal* UNIT, *upon the value of which the values of all the other units in that Table depend.*

The word "YEAR" is derived from the A.S. *gear*, with which is perhaps connected our word "*go*." In Sanskrit the word "*varsha*" signifies "*rain*" and also "*year*," and it seems most likely that our word "year" is derived from this word "varsha." At any rate, in Eastern countries the year is reckoned *from one rainy season to another*. In the West, however, the year was long reckoned from *spring to spring;* in England, for example, the year used to begin on March 25, the vernal equinox. In this way we connect "*year*" with the Latin "*ver*"—*i.e.*, spring.

From this we gather that one of the most primitive divisions of time was into such periods as elapsed between the recurrence of the rainy seasons—*i.e.*, into *years*.

This YEAR having been adopted as the unit, it came afterwards to be divided into MONTHS (*i.e.*, moons), of which *twelve* made up the *year*.

Similarly the *month* was divided into four WEEKS. The word "*week*" is perhaps connected with the Sanskrit VICH, signifying "*to separate*," and so was meant to describe a *distinct* portion of time, especially from Sunday to Sunday. Perhaps, however, it would be more correct to consider the *week* as a collection of *days* rather than as a division of the

month. The word "DAY" is probably derived from the Sanskrit DIV, signifying "*to shine,*" and thus indicating the period of time elapsing between the successive shinings of the sun or moon—*i.e.*, between noon and noon, or midnight and midnight.

The pupil should now learn the following table by heart:—

60	Seconds (L. *secundus*)*	=	1 Minute.
60	Minutes (L. *minutus*)†	=	1 Hour.
24	Hours (Gr. *hora*)‡	=	1 Day.
7	Days	=	1 Week.
4	Weeks	=	1 Lunar month.
13	Lunar § months, or		
12	Calendar ‖ months, or	=	1 YEAR (*unit*).
365	Days		
100	Years	=	1 Century.¶

35. Of the English Tables of Space.

Coming now to

The English Table of Length,

it will be convenient to consider a FOOT as the principal unit; for this word "*foot*" signifies literally "*that which goes,*" and its length as a measure was determined by the *length of a man's foot.*

There was a Saxon word INCE (connected with L. UNCIA), signifying *the twelfth part of a thing,* as of a pound, an acre, &c. From this *ince* our word INCH has been derived, and has been made to signify *the twelfth part of a foot.*

---

* From *sequor*, I follow; cf. consequence.

† From *minuo*, and signifying "made smaller;" cf. diminished, minished, minor, minus.

‡ Originally signifying "any limited period of time," or "a definite space of time fixed by natural laws;" cf. "His *hour* was not yet come." "The *hour* when he began to amend," &c.

§ From L. *luna*, signifying a *moon;* cf. lunatic, lune, lunette, &c.

‖ From L. *calends*, which the Romans applied to the first day of each month on which the holidays were *called;* thus connected with the Greek *kaleo*, to call.

¶ From L. *centum*, a hundred; cf. centurion, centenarian, centipede, &c.

Another Saxon word, *gyrd*, signified a rod, a switch; from this comes our word "YARD," signifying *a rod used for measuring*, and the length of which is equal to 3 *feet*.

For longer distances we have the MILE from the Latin "*mille* passum*," a thousand paces, each of 5 feet, among the Romans. Our English mile contains 5280 feet—*i.e.*, 1760 yards. The Table of Length is sometimes called—

The Table of Lineal† Measure.

3 Barleycorns	=	1 Inch (1 in.)
12 Inches	=	1 Foot (1 ft.)
3 Feet	=	1 YARD (1 yd.)
5½ Yards	=	1 Rod, Pole, or Perch (1 p.)
40 Perches (220 yds.)	=	1 Furlong (1 fur.)
8 Furlongs (1760 yds.)	=	1 Mile.
3 Miles	=	1 League.
69⅓ Miles, or 60 Geographical Miles	=	1 Degree.

Surface, or **Superficial Area**, is measured by means of *squares*. A piece of wood, a foot long and a foot broad, is called a "*square foot*," so that if we wished to describe the amount of timber required for the floor of a room, we should say it was *so many* "*square feet*."

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* Cf. *Milleped*, *millennium*, *millenarian*, &c.

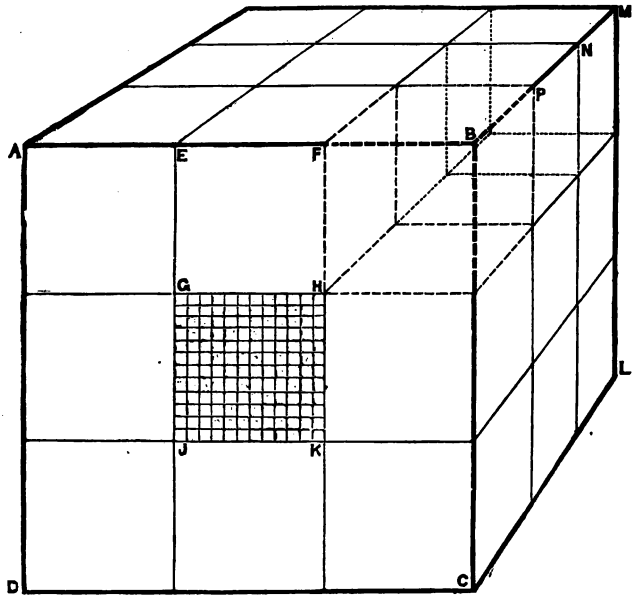
† From L. *linea*, a line.

In teaching these tables the teacher would do well to proceed somewhat as follows: 1. Explain what a *foot* is. 2. Cut off a piece of a rod 1 foot long to exemplify its length. 3. Produce a *yard* measure, and ask the children to guess how many feet it contains. 4. Let one child measure it with the foot-rod, and thus find it contains 3 feet. 5. Explain that 3 feet = 1 *yard*, here giving the name "yard." 6. Similarly show that 12 inches = 1 foot. 7. To more firmly impress these facts, set such mental sums as the following: "*If a yard of calico cost* 1s., *what was the price per foot?*" "*What is the cost of* 1 *foot of silk at* 3d. *an inch?*" &c. 8. When the children are thoroughly acquainted with the number of feet in a yard, the number of inches in a foot, &c., let the whole table be drawn up on the Black-board, copied by the children, and learnt by heart.

N.B.—The explanations given in the text do not pretend to describe absolutely the order in which the units were adopted, for that is open to much dispute; all that is here endeavoured is *to make the learning of these tables as interesting as possible*. With the history of the units further than this, we have no concern.

Similarly, **Solidity**, or **Cubic Content**, is measured by means of solids, each face of which is a square. Such a solid is called a *Cube*, so that solidity is estimated in *cubic inches*, *cubic feet*, &c.

In the accompanying diagram, let AB represent a *yard* divided into 3 *feet*, of which AE is one, EF is another, and FB is the third.



Then GH also is a foot, and is divided into 12 parts representing *inches*.

Let ABCD be a square; then it represents a *square yard* divided into 9 *square feet*, of which GK is one, and BH another. It is evident that the *square foot*, GH, is divided into 144 small squares representing *square inches*.

Hence we gather that—

(1.) 1 sq. yd. = (3 × 3) sq. ft., *i.e.*, "3 *squared*"* sq. ft. = 9 sq. ft.
(2.) 1 sq. ft. = (12 × 12) sq. in., *i.e.*, "12 *squared*" sq. in. = 144 sq. in.

If the whole figure represent a *cubic yard* of which AL is the diagonal, then it is clear that if BM be divided into three equal parts by the points N, P, then BP is one linear foot, HP is the diagonal of a cubic foot, and the cube AL contains (3 × 3 × 3, *i.e.*,) 27 such cubic feet.

It is also clear that upon each of the squares in GK a small cube, called a *cubic inch*, could be placed, and that the cube HP contains (12 × 12 × 12, *i.e.*,) 1728 such *cubic inches*.

Hence we gather that—

(1.) 1 cub. yd. = (3 × 3 × 3) cub. ft., *i.e.*, "3 *cubed*" cub. ft., *i.e.*, 27 cub. ft.
(2.) 1 cub. ft. = (12 × 12 × 12) cub. in., *i.e.*, "12 *cubed*" cub. in., *i.e.*, 1728 cub. in.

The Table of Square Measure (or Measure of Surface).

144 Square Inches (sq. in.)	=	1 Square Foot (1 sq. ft.)
9 Square Feet	=	1 SQUARE YARD (1 sq. yd.)
30¼ (*i.e.*, 5½ *squared*) Square Yards	=	1 Square Pole (1 sq. po.)
40 Poles	=	1 Rood.†
4 Roods (or 4840 sq. yds.)	=	1 ACRE.‡

The Table of Cubic Measure (or Measure of Solidity).

1728 Cubic Inches (cub. in.)	=	1 Cubic Foot (1 cub. ft.)
27 Cubic Feet	=	1 CUBIC YARD (1 cub. yd.)

36. Of the English Table of Capacity.

Although it is possible to estimate wine, beer, and other liquors by the table of Cubic Measure, it has been found more convenient to estimate

* *A number squared* is the product found by multiplying that number by itself once, thus, *five squared* = 5 × 5 = 25.
† Literally, *a rod*; so called from the rod used in measuring.
‡ Literally, *a field*. Cf. L. "*ager*," and our "*agriculture*."
The *acre* is the unit usually employed in estimating the larger areas; the *square yard* is generally employed for the smaller areas.

them by another table called *the Table of Capacity*, the value of whose principal unit is determined by that of the unit of Cubic Measure. This unit of capacity is the GALLON. By this measure also corn, potatoes, &c., are measured.

The Table of Capacity.*

4 Gills or Noggins	=	1 Pint (1 pt.)—"A *painted* or marked measure."
2 Pints	=	1 Quart (1 qt.)—"A *quarter* of a gallon."
4 Quarts	=	1 GALLON (1 gal.)—From old Fr. *galon*, a bowl.
2 Gallons	=	1 Peck (1 pk.)—Cf. "*pack*."
4 Pecks	=	1 Bushel (1 bush.)—Lit. "*a little box*."
8 Bushels	=	1 Quarter (1 qr.)—*The quarter of a ton of grain.*
5 Quarters	=	1 Load.

N.B.—36 Gall. = 1 Barrel.
54 Gall. of Beer, 63 Gall. of Wine } = 1 Hogshead.

37. Of the English Table of Weight.

Metals, and minerals, and some other things, are usually estimated by the following—

Tables of Weight.

1. Troy Weight.

24 Grains (grs.)	=	1 Pennyweight (1 dwt.)
20 Pennyweights	=	1 Ounce (1 oz.)
12 Ounces	=	1 POUND (1 lb.)

2. Avoirdupois† Weight.

16 Drams (drs.)	=	1 Ounce (1 *oz.*)—"*The twelfth*," see "*Inch*."
16 Ounces	=	1 POUND (1 *lb.*)—Cf. L. *pondus*, weight; from *pendo*, to hang.
28 Pounds	=	1 Quarter (1 *qr.*)—*i.e., of a Hundredweight.*
4 Quarters	=	1 Hundredweight (1 *cwt.*), or 112 lb.
20 Hundredweights	=	1 TON.

N.B.—1 *lb. Av.* = 7000 *grains Troy.*

---

* Cf. Capacious, capacitate, incapacitated, from L. *capio*, to take or seize.

† From the Fr. "*avoir du pois*," to have weight.

38. Of the English "Cloth Measure."

2¼ Inches	=	1 Nail.	
4 Nails	=	1 Quarter.	
4 Quarters	=	1 YARD.	
5 Quarters	=	1 English Ell.—From A.S. *eln*, the arm, because originally measured by the arm.	

II. OPERATIONS IN "WEIGHTS AND MEASURES."

39. Of Reduction.

In "*Division of Money*" we often found it necessary to convert sums of money expressed in terms of *several* denominations into an equivalent sum expressed in terms of *one* denomination only, as when £5, 16s. was expressed as 116s. In "*Multiplication of Money*," on the other hand, we sometimes had to perform quite the contrary operation, as when 116s. was expressed as £5, 16s.

In all operations in "Weights and Measures" we meet with cases of *conversions* similar to these, and the process by which the conversion is effected is called **Reduction.**

As may be seen from the above examples, Reduction is of two kinds, viz.—

1. *To terms of a lower denomination.*

   Ex. Reduce £3, 17s. to shillings.

2. *To terms of a higher denomination.*

   Ex. Reduce 306s. to pounds and shillings.

40. Of the Rule for Reduction to terms of a lower denomination.

Ex. 1. *Reduce* £3, 6s. 7d. *to pence.*

Here £1 = 20s.; therefore £3 = (3 × 20)s. = 60s.
And £3, 6s. = £3 + 6s.; therefore £3, 6s. = (60 + 6)s. = 66s.
Again 1s. = 12d.; therefore 66s. = (66 × 12)d. = 792d.
And £3, 6s. 7d. = [£3, 6s.] + 7d. = 66s. + 7
Therefore 66s. 7d. = (792 + 7)d. = 799d.

Answer, **799**d.

Upon such examples as this we base the following

RULE FOR REDUCING QUANTITIES TO A LOWER DENOMINATION—

Multiply the term in the highest denomination by the number which expresses how many times the unit of this denomination contains the unit in the next lower denomination, and add to the product such units of the lower denomination as may be included in the given sum.

Repeat this operation till the given quantity is expressed in terms of the required denomination.*

Ex. 2. *Reduce, by the rule,* 4 yd. 2 ft. 7 in. *to inches.*

4 yd. 2 ft. 7 in.
3
12 ft. + 2 ft. = 14 ft.
12
168 in. + 7 in. = 175 inches.

1st, Multiply 4 by 3 and add 2 to the product, thus obtaining 14.

2nd, Multiply this 14 by 12, and add 7 to the product, thus obtaining 175.

Answer, **175** inches.

A more compendious form of stating this would be—

4 yd. 2 ft. 7 in.
3
14 ft.
12
175 in.

Here the 2 ft. are added to the 12 ft. as the multiplication proceeds. In the same way, the 7 in. are added, thus—

$(4 \times 12) + 7 = 55$; set down 5 and carry 5.
$(1 \times 12) + 5 = 17$; set down 17.

41. Of the Rule for Reduction to terms of a higher denomination.

Ex. 3. *Reduce* 720 *pence to shillings.*

(1.) This question might be set thus—

"*How many shillings are there in* 720 *pence?*"

Now, 12 *pence make* 1 *shilling*, so that we want to know

"*How many times* 12 *pence is contained in* 720 *pence?*"

And § 28 teaches us that this can be found by dividing 720 by 12.

Now, 720d. ÷ 12d. = 60. ∴ *Answer*, **60** shillings.

---

* Before enunciating the rule, the teacher should lead his pupils to look into the working of *several* easy examples treated in the same manner that we have treated Ex. 1. above; and *the rule* should be obtained from the children by the teacher's inductive questioning upon these examples.

Ex. 4. *Reduce* 1723 *farthings to £, s. d.*

Since 4f. = 1d., we have

$$1723\text{f.} = \frac{1723}{4}\text{d.} = 430\text{d. } 3\text{f.} = 430\tfrac{3}{4}\text{d.}$$

And, 12d. = 1s., therefore

$$430\text{d.} = \frac{430\text{s.}}{12} = 35\text{s. } 10\text{d.}$$

$$\therefore\ 430\tfrac{3}{4}\text{d.} = 35\text{s. } 10\tfrac{3}{4}\text{d.}$$

And, 20s. = £1, therefore

$$35\text{s.} = \frac{£35}{20} = £1,\ 15\text{s.}$$

$$\therefore\ 35\text{s. } 10\tfrac{3}{4}\text{d.} = \mathbf{£1,\ 15s.\ 10\tfrac{3}{4}d.},\ \textit{Answer.}$$

From an observation of the method pursued in these two examples, we arrive at the following

RULE FOR REDUCING QUANTITIES TO A HIGHER DENOMINATION:—

Divide the given quantity by the number which expresses how many units of the given denomination make a unit of the next higher denomination.

Repeat this operation till the given quantity is expressed in the required denomination.

In each case the remainder (if any) will be of the same denomination as the dividend from which it arises, and will be so expressed in the answer.

Ex. 5. *Reduce, by the Rule,* 1645 lb. *to cwt. qr. lb.*

(1.) Divide by 28—*i.e.*, 7 × 4. The 3 over is 3 seven pounds—*i.e.*, 21 lb.

(2.) Divide by 4; the 2 over is 2 qr.

```
28 { 7 | 1645 lb.
     4 | 235 times 7 lb.
         58 times 28 lb. + 3 times 7 lb. =
       4 | 58 qr. 21 lb.
           14 cwt. 2 qrs. 21 lb., Answer.
```

Examples in both kinds of Reduction.*

Ex. 6. *Reduce* 31 fur. 4 yd. *to feet, and prove the answer.*

31 fur. 0 poles, 4 yd.
40
1240 poles.
$5\frac{1}{2}$
6200 = 5 times 1240
620 = $\frac{1}{2}$ of 1240
yd. 6820 = $5\frac{1}{2}$ times 1240
Therefore,
6824 yd. = 6820 yd. + 4 yd.
3
20472 ft., *Answer.*

PROOF.

3 | 20472 ft.
$5\frac{1}{2}$ | 6824 yd.
2
11 | 13648 half yd.
1240 poles 8 half yd.
4,0 | 124,0 poles 4 yd.
31 fur. **0** poles **4** yd.

Ex. 7. *Reduce* 463182 sq. ft. *to acres, roods, &c., and prove the answer.*

9 sq. ft. | 463182 sq. ft.
51464 sq. yd. 6 sq. ft.

We want now to reduce 51464 sq. yd. to perches; this is done by finding how many times $30\frac{1}{4}$ sq. yd. is contained in 51464 sq. yd.—*i.e.*, how many times ($30\frac{1}{4} \times 4$, *i.e.*), 121 *quarters of a sq. yd.* are contained in (51464×4, *i.e.*), 205856 *quarters of a sq. yd.*

We find, by Division, that 121 is contained 1701 times in 205856, and 35 are "over." This 1701, then, represents **1701** perches, and the 35 *over* signifies **35** quarters of a sq. yd.—*i.e.*, 35 sq. yd. ÷ 4—*i.e.*, 8 sq. yd. 6 sq. ft. 108 sq. in.

∴ 51464 sq. yd. = 1701 perches 8 sq. yd. 6 sq. ft. 108 sq. in.

Therefore 463182 sq. ft.—*i.e.*,

51464 sq. yd. 6 sq. ft. = 1701 perches 8 sq. yd. (6 + 6, *i.e.*,) 12 sq. ft. 108 sq. in.
= 1701 perches 9 sq. yd. 3 sq. ft. 108 sq. in.

It now remains *to reduce* 1701 *perches to roods*, &c.

4,0 | 170,1 perches
4 | 42 roods 21 perches
10 acres 2 roods 21 perches.

Therefore, the whole *Answer is*, **10** ac. **2** roods **2** p. **9** sq. yd. **108** sq. in.

* The teacher should point out that this Rule called "*Reduction*" has nothing to do with "*Diminution.*" *To Reduce* here simply means "*otherwise to express,*" "*to convert into an equivalent.*"

The whole process of working is now exhibited below, together with the proof.

```
        9 | 463182 sq. ft.
      30¼ | 51464 sq. yd. + 6 sq. ft.
          |     4
 ---------------------------------------------------------------------
 121 { 11 | 205856
     { 11 | 18714 + 2 } 35 qrs. of a sq. }   = 8 sq. yd. 6 sq. ft. 108 sq. in. +
        40 | 1701 + 3 }   yd. + 6 sq. ft. }     6 sq. ft.
          4 | 42 roods 21 perches          = 8 sq. yd. 12 sq. ft. 108 sq. in.
             10 acres 2 roods.             = 9 sq. yd. 3 sq. ft. 108 sq. in.
```

Answer, **10** acres **2** roods **21** perches **9** sq. yd. **3** sq. ft. **108** sq. in.

Reduce 10 ac. 2 r. 21 p. 9 sq. yd. 3 sq. ft. 108 sq. in. *to sq. ft.*

```
                4
               42 roods
               40
              1701 perches
                30¼
              51030 sq. yd.                  = 1701 sq. yd.* × 30
                425 sq. yd. 2 sq. ft. 36 sq. in. = 1701 sq. yd.* ÷ 4
                  9 sq. yd. 3 sq. ft. 108 sq. in. = Remainder of the given
By Addition, 51464 sq. yd. 6 sq. ft.                quantity.
                  9
             463182 sq. ft., Answer—the original quantity.
```

Exercise 16.

Exercises in Reduction.

1. Reduce (proving each result) £3, 2s. 6d. to pence; £100, 7s. 3d. to pence; £16, 14s. 1½d. to farthings; 18s. 0½d. to half-pence; £25 to farthings; 101 guineas to half-pence.

2. How many *fourpences* are worth £2, 17s.? and how many *threepences* in 9 times that sum?

---

* It might at first be supposed that this ought to be PERCHES. Consider, however, thus—

1 *perch contains* 30¼ sq. yd.

∴ 1701 perches contain (30¼ × 1701) sq. yd. = (1701 × 30¼) sq. yd. = 1701 sq. yd. × 30¼.

3. How many half-crowns are equal in value to the sum of 31 five-pound notes, 26 sovereigns, 18 crowns, and 40 threepences? and how many fourpenny buns could be purchased with them?

4. How many *guineas* would be worth as much as nine hundred and three sovereigns.

5. How many baskets of oranges, at half-a-crown a basket, are worth 75 barrels of beer at half-a-guinea each?

6. Reduce 2703 pence to £, s. d.; 82071 farthings to £, s. d.

7. Reduce (proving each result) 71 yd. 2 ft. 7 in. to inches; £3, 5s. to farthings; 5 days 7 hrs. 30 min. to seconds; 13 tons 27 lb. to lb.; 13 cwt. 2 qr. 13 oz. to drs.; 13 half-guineas to sixpences.

8. Reduce 14 fur. 2 p. 4 yd. to inches; 2 ac. 3 r. 10 sq. yd. to sq. in.; 215 roods 103 sq. in. to sq. in.; 171 cub. yd. to cub. in.

9. Convert 3013 hrs. into wks. and hrs.; 17000 seconds into wks. days and hrs.; 91738 days into lunar months, wks. and days.

10. Express 2 million linear inches in terms of higher denominations; also 21073 ounces *Av.*, and the same number *Troy*.

11. How many glasses of wine, each containing 2 gills, can be filled from a hogshead from which 2 quarts have leaked?

12. After a publican had sold 858 half pints of ale from a hogshead, he found it was empty. Had he drawn out more or less than the hogshead ought to have contained, and how much?

13. How many glass marbles, each weighing half an ounce, could be made from a ton and a half of glass? and what would they be worth at 2 for a farthing?

14. A reel of cotton contains 255 yards, how many times will it be wound round the reel which is one inch and a quarter in circumference?

15. How many weeks, days, &c., will it take a clock to tick 2786400 times if it ticks 43 times a minute?

16. A patchwork quilt contained 1156 stars, and each star contained 7 small hexagons. The size of two such hexagons being 1 nail (*cloth measure*), express in ells, qr., and inches the size of the quilt.

17. How much dishonest profit would a publican make in a week supposing that he served 213 persons a day with a quart of beer each, the measure being 1 gill short in every three quarts, and the price of a quart of beer being 5d.?

18. When the Income-tax was 3d. in the pound, a clerk found his salary for the year was reduced by £38, 17s. 3d.; what must his annual income have been?

42. Of Compound Addition and Multiplication.

The reasons of the following rules have been sufficiently explained in Arts. 19–32.

(1.) RULE FOR COMPOUND ADDITION—

Having arranged the given quantities, so that in each case the terms of the same denomination form a vertical column, find the sum of the numbers in the right hand column, reduce the sum to terms of its own and the next higher denomination, set down the units of the present denomination beneath its own vertical column, and add the units of the higher denomination to the sum of the units in the next vertical column.

Proceed in the same way with all the columns.

(2.) RULE FOR COMPOUND MULTIPLICATION—

Having placed the multiplier under the lowest term of the multiplicand, multiply the given units in this term by the multiplier, and, having "reduced" the product as in Compound Addition, put down the units of this denomination and carry the units of the higher denomination to the next product.

Repeat this operation with each term of the multiplicand.

Ex. *Add together* 31 yd. 2 ft. 3 in.; 17 yd. 1 ft. 10 in.; 2 ft. 7 in.; 101 yd. 9 in.; *multiply the sum by* 8, *and reduce the product to inches.*

	yd.	ft.	in.
	31	2	3
	17	1	10
		2	7
	101	0	9
By Addition,	**151**	**1**	**5**
			8
By Multiplication,	**1211**	**2**	**4**

yd.	ft.	in.
1211	2	4
3		
3635 ft.		
12		
43624 in.		*By Reduction.*

Answer, **43624** in.

N.B.—The latter part of this sum might have been worked thus—

151 yd. 1 ft. 5 in. = 5453 in. *By Reduction.*
8
By Multiplication, **43624** in. *Answer.*

43. Of Compound Subtraction and Division.

(1.) RULE FOR COMPOUND SUBTRACTION *—

Having placed the less number below the greater, in such a manner that units of the same denomination may be in the same vertical column, subtract, if possible, the subtrahend in the right-hand column from the corresponding minuend, and set the remainder beneath the subtrahend. If, however, the subtrahend exceed the minuend in any denomination, add to this last as many units as form one unit of the next higher denomination, from the sum take the subtrahend, and set down the remainder, carrying ONE to the next subtrahend.

Repeat the process with each vertical column, proceeding always from *right* to *left*.

(2.) RULE FOR COMPOUND DIVISION—

Arrange the divisor and dividend as in Simple Division; then—

***a. If the divisor be a concrete quantity*, reduce both it and the dividend to terms of the lowest denomination which is contained in either of them, and then divide as in Simple Division; but—**

***b. If the divisor be an abstract number* find how many times the number in the highest denomination of the dividend contains the divisor, set the result in the quotient, and reduce the remainder and the number in the next denomination lower, to units of this lower denomination, and perform the same process as with the last denomination, and proceed similarly with the remaining terms of the dividend.**

Ex. 1. *Find the difference between* 12 tons 1 qr. 4 lb. *and* 13 tons 17 cwt. 1 qr. 16 lb. 7 oz.; *and find how many times* 2 qr. 15 oz. *can be taken from the answer.*

	tons.	cwt.	qr.	lb.	oz.
	13	17	1	16	7
	12	0	1	4	0
By Subtraction,	1	17	0	12	7

By Reduction, 1 ton 17 cwt. 12 lb. 7 oz. = **66503** oz.
and 2 qr. 15 oz. = **911** oz.
Now 66503 ÷ 911 = **73**. *Answer.*

* It should be noticed that these rules describe strictly the processes not only of the *Compound* but also of the *Simple* Rules, where, instead of dealing with yards, feet, inches, &c., we deal with hundreds, tens, units, &c.

Ex. 2. *Multiply* 2 m. 3 fur. 7 p. 3 yd. *by* 1031, *and prove the result.*

	m.	fur.	po.	yd.		
	2	3	7	3		
				10		
	23	7	35	$2\frac{1}{2}$	=	10 times.
				10		
	239	6	34	3	=	100 times.
				10		
	2398	4	25	$2\frac{1}{2}$	=	1000 times.
	71	7	26	2	=	30 times.
	2	3	7	3	=	1 time.
By Multiplication,	**2472**	**7**	**19**	**2**	=	1031 times. *Answer.*

PROOF. *By Division.*

```
        m.  fur. po. yd.
1031) 2472  7   19  2 (2 m. 3 fur. 7 po. 3 yd.
      2062                 The original multiplicand.
      ----
       410
         8
      ----
      3287
      3093
      ----
       194
        40
      ----
      7779
      7217
      ----
       562
       5½
      ----
      2812
       281
      ----
      3093
      3093
      ----
```

Ex. 3. *Find the difference between* 21 fur. 1 ft. 7 in. *and* 3 fur. 7 po. 5 yd. 2 ft. 1 in.

21 f.	0 po.	0 yd.	1 ft.	7 in.
3 f.	7 po.	5 yd.	2 ft.	1 in.
17 f.	**31** po.	**5** yd.	**2** ft.	**6** in.

In the yards, we have 5 in the subtrahend, and 0 in the minuend. Borrowing 1 pole, we have then 6 in the subtrahend and $5\frac{1}{2}$ in the minuend. In such a case as this it is better to borrow **2** poles (instead of 1) thus obtaining 11 in the minuend. After subtracting, 2 (instead of 1) must be carried.

Exercise 17.

Exercises in Compound Addition.

1. Find the sum of £80, 13s. 2¼d., £2001, 10s., £7004, 0s. 6d., 19s. 0¼d., £1, 7s. 10d., £91621, 3s. 1¾d.

(2.)			(3.)			(4.)			(5.)			
oz.	dwt.	gr.	oz.	dwt.	gr.	tons.	cwt.	qr.	tons.	cwt.	lb.	oz.
21	3	10	701	16	14	1017	13	2	73	12	90	10
9	17	21	8	0	19	840	16	3	205	17	100	13
16	9	3	5670	12	5	9	4	0	17000	3	84	0
805	10	17	39	17	10	5000	17	1	916	10	31	15
30	19	20	140	13	16	91	0	2	2018	19	0	11
6	18	13	10	0	0	415	0	2			103	7

6. How many seconds are there in the sum of 25 hr. 4 min., 3 days 5 hr. 2 min., 31 min. 16 sec., 1 week 1 day 3 hr. 2 sec.?

7. What is the total length of a railway which is divided into seven distinct parts of the following lengths—12 miles 5 fur., 30 miles 7 fur. 13 po., 19 miles 17 po., 28 miles 5 fur. 2 yd., 21 fur. 19 po., 163 fur., 7 fur. 19 po., and 201 miles 3 fur. 30 po.?

(8.)			(9.)			(10.)				(11.)			
yd.	ft.	in.	ac.	r.	po.	ac.	r.	po.	yd.	lb.	oz.	dwt.	gr.
13	2	7	203	1	9	1361	2	17	3	103	10	19	17
400	1	10	70	3	12	970	0	16	28	64	3	0	21
1621	0	9	149	0	38	28	3	29	17	2018	11	19	16
3	2	11	76	0	19	500	1	27	14	644	9	3	23
5	0	3	465	3	27	48	2	36	17	20	10	17	16
16	0	4	92	2	16		2	0	6	109	10	3	22

(12.)					(13.)						(14.)		
tons.	cwt.	qr.	lb.	oz.	lea.	m.	fur.	yd.	ft.	in.	sq. yd.	sq. ft.	sq. in.
19031	3	1	6	0	27	1	7	2	0	10	1700	3	100
72	17	0	23	10	301	2	3	0	1	11	2083	7	43
109	16	2	18	7	38	0	6	0	1	11	907	2	67
4570	9	0	0	13	28	0	0	4	2	0	13017	6	93
28	0	3	20	14	199	2	7	5	1	7	48	3	107
516	12	2	18	10			3	2	1	3	707	8	31
27	14	0	10	12		2	2	3	0	10	43	5	89
	13	1	17	12	2000	1	5	1	2	9	76808	0	143

15. How old was a man, just discharged from prison, who was 6 yr. 5 months old when sent first to school, remained there 3 yr. 9 months, and then disappeared for 1 yr. and a half, at the end of which time he was sent to a reformatory where he stayed 7 yr. and 7 months, and leaving that institution was convicted of several robberies and sentenced to a continuous imprisonment of 3 months for a petty theft, and 4 yr. for burglary?

16. At a tea-party there were 4 cakes made, weighing respectively 7 lb. 10 oz. 11 dr., 13 lb. 5 oz. 9 dr., 12 lb. 10 oz., and 11 lb. 13 oz. 7 dr., what was the weight of all that cake?

17. There were also 5 urns of tea used, one urn contained 5 quarts, the second contained 1 quart 1 pint more than the first, the third contained as much as the two first together, the fourth as much as the second and third together, and the other contained 2 quarts. How much tea was provided for that party?

Exercise 18.

Exercises in Compound Multiplication.

1. Multiply £17649, 13s. 7¼d. by 9, 64, 13, 59, 101, 177, 3006, 17403.
2. Multiply 18 tons 7 cwt. 16 lb. 3 oz. by 11, 7, 36, 91, 108, 448, 1817.
3. Multiply 27 lb. 5 oz. 19 dwt. 17 gr. by 13, 19, 29, 27, 46, 107, 703, 2408.
4. Multiply 6 yd. 2 qr. 2 nails by 47, 59, 64, 18, 576, 4061.
5. Multiply 27 wk. 3 days 19 hr. 25 min. by 18, 907, 4376, 80079.
6. Multiply 183 bush. 3 pk. by 72, 81, 65, 85, 390, 706, 5080.
7. Multiply 7 gal. 3 qt. 1 pt. by 96, 108, 121, 270030, 16007.
8. Multiply 4 ac. 2 r. 17 po. by 48, 87, 69, 509, 7017, 8080.
9. Multiply 17 sq. yd. 2 sq. ft. 93 sq. in. by 10, 807, 276, 14309.
10. Multiply 208 cub. yd. 10 cub. ft. 191 cub. in. by 1603, 28071.
11. Multiply 17 lea. 2 m. 3 fur. 12 po. 2 yd. by 60, 19, 2800, 5670.
12. If 5 lb. 12 oz. of bread be served out to each man in an army of 18000 men, what amount of bread would be required?
13. When the Prince of Wales was married, a school of 341 children were supplied with *favours* by their clergyman; if each favour contained 1 yd. 7 in. of ribbon, how much ribbon was consumed in making a sufficient number of favours to supply each child with one?
14. A gentleman gave 2 cwt. 3 qr. of coal to each poor person in his parish at Christmas. If 50 men and 47 women received his gift, how much coal did he give away? and what was its value at 1s. 3d. per cwt?

15. How much cloth will be required to make suits of clothes for 1500 soldiers if each suit contains 5 yd. 3 qr. 2 nails?

16. An estate is cut up into lots for building houses, provided each with 1 rood 13 poles 19 sq. yd. of garden. Each house stands on 22 poles 17 sq. yd. of ground, and has also, beside the garden, a court containing 7½ poles of ground. It was found that 399 such houses, each with its garden and court, could be arranged out of the estate; how large was it?

17. What is the weight of coal in a train which consists of 17 trucks, each truck containing 9 tons 17 cwt. 47 lb.? and what is the total weight the engine has to move, supposing that, on an average, each truck weighs 4 tons 15 cwt. 107 lb., and the engine and tender 23 tons 17½ cwt.?

Exercise 19.

Exercises in Compound Subtraction.

1. Find the difference between £1737, 10s. 4d. and £1800, 1s.

2. How much remains when £706, 19s. 0½d. has been taken from £1300?

3. By how much does 101 yd. 2 ft. 7 in. fall short of 230 yd. 3 in.?

4. How much must be added to 47 ton 12 cwt. 1 qr. 16 lb. to make it 91 tons 3 qr.?

5. By how much does 140 lb. 5 oz. 3 dwt. 4 gr. exceed 29 lb. 10 oz. 10 dwt. 10 gr.?

6. How many cubic inches must be added to 3 cub. yd. 7 cub. ft. 39 cub. in. to make it 19 cub. yd. 2 cub. ft. 13 cub. in.?

7. The area of a parlour being 106 sq. yd. 91 sq. in., by how many sq. in. is it less than that of a drawing-room whose area is 213 sq. yd. 3 sq. ft.?

8. One piece of land containing 17 acres 3 r. 14 po. 5 yd. was exchanged for a sum of money and 9 acres 3 r. 27 po. 6 yd. of land. For how much land did the money exchange?

9. From 21 tons 3 cwt. 96 lb. take 14 tons 17 cwt. 3 oz. and reduce the result to ounces.

10. Multiply 3 days 21 hr. 14 sec. by 73, and subtract the product from a year of 365 days.

11. If a horse and carriage be together worth 104½ guineas, of which the horse is worth 783 half-crowns, what is the value of the carriage?

12. There are two blocks of granite, one containing 23 cub. yd. 14 cub. ft., the other being smaller; if the difference between their sizes is 94631

cub. in., what is the size of the smaller one? and how many cub. in. of granite are contained in the two together?

13. The total length of 4 pieces of ribbon is 309 yd.; if one piece contained 17 yd. 3 nl. 1 in., a second 103 yd. 2 qr. 2 nl., a third 23 yd. 2 in., what was the length of the other?

14. Multiply 7 qr. 6 bush. 3 pk. by 41; increase the product by 191 qr. 3 bush., and subtract the sum from 650 qr. 1 pk.

15. If the price of 17 cwt. 39 lb. of coal be 20s., how much coal was purchased for £89? If one waggon carrying 4 tons 17 cwt. never reached its destination, how much of this coal was actually delivered?

16. Find the value of—

a. (2 tons 10 cwt. 1 qr. – 17 cwt. 17 lb. 1 oz.) × 19.
b. (13 lb. 13 oz. 11 dr. – 3 lb. 13 dr.) × 47.
c. (27 lb. 14 dwt. – 19 lb. 3 oz. 16 dwt. 4 gr.) × 9013.
d. 60 qr. 5 bush. 1 gall. – (2 bush. 1 gall. × 73).
e. (208 days 14 min. 3 sec. × 793) – (25 days 17 sec. × 4019).
f. The difference between (3 ac. 2 r. 27 po. 27 sq. yd. 4 sq. in. × 305) and 1000 acres + 1000 roods + 2000 sq. yd. + 1700 sq. in.
g. (29 ells 3 qr. 3 nl. 2 in. × 51) reduced by 9 ells 2 qr. 2 nl., after being increased by 20 ells 4 qr. 3 nl. 2 in.

Exercise 20.

Exercises in Compound Division.

1. Divide £17614, 0s. 9¾d. by 7, 9, 32, 65, 87.
2. Divide 57974 yd. 1 ft. 9 in. by 3, 7, 5, 11, 13.
3. Divide 177458 t. 10 cwt. 14 lb. by 9, 6, 7, 13, 41.
4. Divide 27081 ac. 1 r. 27 po. 19 sq. yd. by 7, 5, 31, 29, 605.
5. Divide 5081 gal. by 79, 43, 62, 9009.
6. Divide 31 yr. 149 days by 703, 51, 62, 84.
7. Divide 2000 Eng. ells 3 qr. 1 nl. by 75, 89, 307.
8. A party of 5 miners having discovered 73 oz. 12 dwt. 2 gr. of gold, what will be the share of each?
9. A hall, containing 418 sq. yd. 108 sq. in., is paved with 5017 square tiles, what is the size of each tile?
10. A man purchased a house with the savings of three years. The price of the house was £120, and the cost of conveyance came to £3, 3s. 9d. How much must he have saved per day to have laid by exactly enough?

11. The combined weight of fifty-nine blocks of granite being one hundred and sixty-five tons, what is the weight of each?

12. A field of 3 acres and a half being divided into 38 equal allotments, how much land is contained in 17 of them?

13. A boy, leaving home for school, received from his father £3, 4s. $1\frac{1}{4}$d. Out of this he had to pay 7s. 10d. for travelling expenses, and then had sufficient remaining to provide him with $9\frac{1}{4}$d. per day for pocket money. How many days was he at school?

14. If a furnace consumes 1 ton 17 cwt. 21 lb. of coal per week, how many days will it take to consume 200 t. 16 cwt. 28 lb.?

15. Two railway stations are exactly 17 miles apart, and it is found that between them there are 680 telegraph poles; what is the average length of wire between these poles?

16. How many workhouse girls could be supplied with new dresses from 278 yd. 2 qr. 3 nl. $1\frac{3}{4}$ in. of print, if 5 yd. 3 qr. 2 nl. 2 in. are required for each dress?

17. How many times may 23 bush. 3 pk. 3 qt. be taken from 167883 bush. 3 pk. 3 qt.?

18. How many gallons of rum, at 7s. $3\frac{1}{2}$d. each, can be purchased with £33, 3s. $6\frac{1}{2}$d.?

19. If 13 lb. of sugar be worth 1 lb. of tea, how many lb. of tea are worth as much as 39 casks of sugar, each containing 3 qr. 19 lb.?

20. By what number must 31 miles 3 fur. 99 yd. be multiplied to make it 2420 miles 1 fur. 143 yd.?

CHAPTER IV.

ELEMENTARY MENSURATION.

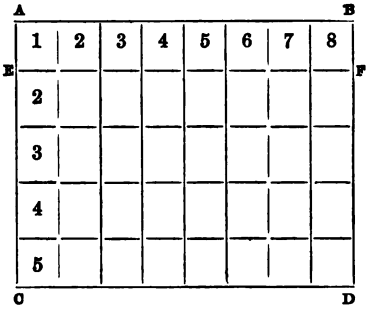
44. Of the Mensuration of Plane Rectangular Surfaces.

It has been explained (§ 35) that surface is usually estimated in *square yards, square feet*, &c. Now, there are two ways of finding the number of square yards, &c., that are contained in any given area, viz.—

1st. By actual measurement with *a square yard* or some other unit of Square Measure; and,

2nd. By finding *the product of its length and breadth.*

Ex. *A piece of carpet is* 8 yards *long and* 5 yards *wide; how many square yards of carpet does it contain?*



Let AB represent 8 yards, and AC 5 yards.

Then,

(1.) *By actual counting of the squares*, we find that there are (8 × 5, *i.e.*), 40 squares; in a word the carpet contains 40 square yards.

(2.) *By considering* that the whole carpet, ACDB, may be divided into FIVE equal strips, of which AEFB is one, each strip containing 8 squares, and each square being *a square yard*—*i.e.*, AEFB contains 8 square yards, and therefore the whole figure ACDB contains (8 square yards × 5, *i.e.*), 40 square yards.

Hence the reason of the following Rule:—

RULE.—To find the area of any regular rectangular* surface, express its *length* and *breadth* in units of one denomination of length, multiply the units in the length by the units in the breadth, and the product is the answer expressed in corresponding units of Square Measure.

Ex. *How many square feet of timber are required for a platform which is* 21 yd. 2 ft. *long and* 10 yd. 1 ft. *wide?*

By the Rule.—*Area* = *Length* × *Width* = 21 yd. 2 ft. × 10 yd. 1 ft. = 65 ft. × 31 ft. = **2015** sq. ft., *Answer.*

N.B.—Since the AREA = LENGTH × BREADTH, it follows that, if any two of these be given, the third may be found, for,

(1.) By the Rule, the AREA = *Length* × *Breadth*;
(2.) The LENGTH, therefore = *Area* ÷ *Breadth*; and
(3.) The BREADTH = *Area* ÷ *Length*.

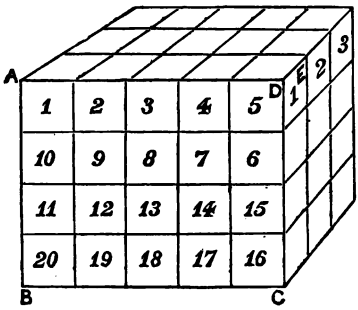
---

* *A Rectangle* is a four-sided figure of which all the angles (*i.e.*, *corners*, from L. *angulus*) are right angles—*i.e.*, such corners as are found in a square.

45. Of the Mensuration of Plane Rectangular Solids.

It has been shown (§ 35) that solidity, or cubic content, is usually estimated in cubic feet and cubic inches. It is not possible, however, always to find the cubic content of a body by actual counting of the number of times it contains a given cube; indeed, the usual method is to measure separately the length, breadth, and thickness of a given body, and from these arrive at its cubic content by finding their continued product.

Ex. *A block of stone is* 5 ft. *long,* 4 ft. *high, and* 3 ft. *thick, how many cub. ft. does it contain?*



Let the accompanying figure represent the block of stone.

Then it is clear that its front face, ABCD, contains (5 × 4, *i.e.*) 20 sq. ft.

It is also clear that, if the block of stone be sawn down through the point E,* the front piece may be again sawn up *into* 20 *small cubes*, having for their respective faces the squares numbered 1 to 20. In a word, the front piece would contain 20 cubic feet of stone.

But it is also evident that the whole block contains THREE such slices of stone as the one we have been considering; therefore the whole block contains (5 × 4 × 3, *i.e.*), 60 cubic feet of stone.

Hence the reason of the following Rule:—

RULE.—To find the content of any regular rectangular solid, express its *length*, *breadth*, and *thickness* as units of the same denomination of length, and find their continued product. The result will be the content of the solid expressed in terms of the corresponding unit of Cubic Measure.

---

* An intelligent teacher might illustrate this to his pupils by a piece of soap, which he might actually cut up into cubes before their eyes.

Ex. *How many cubic feet of air are contained in a room which is* 31 yd. *long,* 15 yd. 2 ft. *wide, and* 19 ft. *high?*

By the Rule.—*Cubic Content* = *Length* × *Breadth* × *Height*
= 31 yd. × 15 yd. 2 ft. × 19 ft.
= 93 ft. × 47 ft. × 19 ft.
= **83049** cub. ft., *Answer.*

N.B.—Since the CUBIC CONTENT = *Length* × *Breadth* × *Thickness*, it follows that, if any three of these be given, the fourth may be found; for

(1.) The CUBIC CONTENT = *Length* × *Breadth* × *Thickness.*

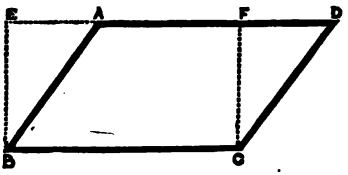
(2.) The LENGTH, therefore $= \frac{\text{Cubic Content}}{\text{Breadth} \times \text{Height}}$*.

(3.) The BREADTH $= \frac{\text{Cubic Content}}{\text{Length} \times \text{Height}}$.

(4.) The HEIGHT $= \frac{\text{Cubic Content}}{\text{Length} \times \text{Breadth}}$.

46. Of the Measurement of Parallelograms and Triangles.

(1.) **A Parallelogram** is a four-sided figure whose opposite sides never meet, though produced ever so far both ways.



From the accompanying figure it is clear that the area of the parallelogram ABCD is the same as that of the rectangle EBCF; hence the reason of the following Rule:—

RULE.—To find the area of a Parallelogram, express its length and perpendicular height in terms of the same unit of length, and find their product, which will be the answer expressed in terms of the corresponding unit of Square Measure.

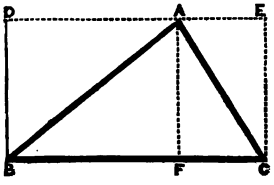
---

* It may be noticed that when either AREA or CUBIC CONTENT are *given*, they are always used as the *dividend*, the remaining given terms being always *divisors*. It is, of course, much better to depend upon an understanding of "*the reason why.*"

Ex. *Find the area of a parallelogram the length of which is* 2 ft. 9 in. *and the height* 1 ft. 7 in.

By the Rule.—Area = *Length* × *Perpendicular Height*
= 2 ft. 9 in. × 1 ft. 7 in. = 33 in. × 19 in.
= **627** sq. in., *Answer.*

(2.) **A Triangle** is a figure bounded by three sides (or, more properly, containing three angles).



In the accompanying figure, it is evident that,

a. The triangle AFC is one-half the rectangle AFCE;

b. The triangle AFB is one half the rectangle AFBD;

And that, therefore,

The whole triangle ABC is one-half the whole rectangle DBCE.

Hence the reason of the following Rule:—

RULE.—The area of a triangle being one-half that of the parallelogram which has the same base and the same height; to find the area of a triangle, reduce its base and height to units of the same denomination of length, multiply the results together and divide the product by 2; the quotient will be the area of the triangle expressed in corresponding units of Square Measure.

Ex. *How many square feet of land are contained in a triangular plot whose base is* 31 yd. 1 ft. *and perpendicular height* 10 yd.?

By the Rule.—Area $= \frac{31 \text{ yd. } 1 \text{ ft.} \times 10 \text{ yd.}}{2} = \frac{94 \text{ ft.} \times 30 \text{ ft.}}{2}$

$= \frac{2820 \text{ sq. ft.}}{2} =$ **1410** sq. ft., *Answer.*

Exercise 21

Exercises in Elementary Mensuration.

1. How many square feet are contained in a room which is 35 feet long and 27 feet wide?

2. How many acres of grass are contained in a rectangular meadow which is 1080 yd. long and 121 yd. wide?

3. In a schoolroom which is 39 yd. wide and 104 yd. long, how many children can be provided each with 8 sq. ft. of space?

4. What is the cost of painting the walls of a room which is 17 yd. long, 15 ft. high, and 5 yd. 2 ft. wide at 3s. per sq. ft., after allowing for a doorway which is 7 ft. high and a yard wide, and for a large window which occupies 38 sq. ft.?

5. How many cub. in. of earth are removed to make a ditch 17 yd. long, 1 yd. and a half wide, and 5 ft. deep?

6. If a cub. yd. of coal weigh 1 cwt., what is the weight of a large block of coal which is 17 yd. long, 8 yd. wide, and 3 yd. and a half thick?

7. If 3 cub. ft. of iron weigh 1 ton, what is the weight of a large mass of iron measuring 13 ft. in length, 4 ft. in thickness, and 1 yd. in width?

8. If a miner can remove 15 cub. yd. of coal per week, how many days will he be occupied in excavating a square cavity whose height is 5 yd., and whose side measures 105 ft.? (*N.B.*—6 days = 1 working week).

9. What is the value of a piece of timber 9 yd. long, 2 yd. wide, and 7 ft. thick, at 1s. 3¼d. per 3 cub. ft.?

10. How many square feet of lead are required for a roof, in the shape of a parallelogram, all of whose sides are of one length, *viz.*, 17 ft., and whose opposite sides are 7 ft. apart?

11. The base of a triangle being 13 ft., and its height 3 ft. 6 in., express its area in sq. in.

12. A corner plot of land contained exactly an acre; the length of its base being 242 yd., what was its width?

13. How many square feet of boards are required for the wainscot of a room which is 29 yd. long and 17 yd. wide, if the wainscot is carried round the room 2 ft. high?

14. How many bottles, each containing 7 cub. in. of water, can be filled from a tank which is 2¼ yd. long, 7 ft. broad, and 5 ft. 8 in. high, and is half filled with water?

15. A pile of bricks was found to be 14½ yd. long, 22 ft. wide, and to contain 45936 bricks. How high was the pile if each brick was 1 ft. long, 5 in. wide, and 3 in. thick?

16. A gentleman went to buy a piece of ground in the shape of a parallelogram. He found that its length was 51 paces, and the owner assured him that it contained exactly 308 sq. yd., which, however, was 162 sq. in. less than its real size. In how many paces could the gentleman step across it if a square, having one of his paces for its side, contains 783 sq. in.?

CHAPTER V.

THE GREATEST COMMON MEASURE AND LEAST COMMON MULTIPLE.

47. Definitions.

In § 20, *N.B.*, it has been noticed that *the product of any two given numbers is the same whichever of them is taken as the multiplier—e.g.*, $6 \times \mathbf{7} = 7 \times \mathbf{6}$; $8 \times \mathbf{5} = 5 \times \mathbf{8}$, &c.

1	2	3	4	5	6	7
2						
3						
4						

This truth may be illustrated by the accompanying diagram, which contains 28 squares—*i.e.*, 4 times 7 squares if counted *horizontally*, or, 7×4 squares if counted *perpendicularly;* from which it follows that 4 *can* SEVEN *times be taken from* 28, and that 7 *can* FOUR *times be taken from* 28 without remainder in either case. We generally express these facts by saying that—

(1.) **Four** *and* **Seven** *are each contained in* 28 *an exact number of times;*

(2.) **Twenty-eight** *contains both* 4 *and* 7 *an exact number of times;* or, **Twenty-eight** *can be exactly divided by* 4 *and by* 7. And it may be added that—

(3.) *When a number will divide another number without remainder, the first is said to be a* MEASURE *of the second, and the second is said to be a* MULTIPLE *of the first.*

Thus, 4 and 7 and 2 are each MEASURES of 28, and 28 is a MULTIPLE of each of these.

(4.) *When one number is a measure of another the first is said* TO MEASURE *the second.*

Thus, 2, 4, 7 each MEASURE 28.

(5.) *When one number will divide each of two or more numbers without remainder, it is said to be a* COMMON * MEASURE *of these numbers.*

Thus, 7 is a measure of 28, of 35, and of 77; it is therefore a COMMON MEASURE of 28, 35, and 77.

* Cf. *Common* property, *common* land, *common* cause, &c.

(6.) *The greatest number which is a common measure of two or more numbers is called their* GREATEST COMMON MEASURE.

Thus, **2**, **3**, and **4** are each *common measures* of 144 and 360, but **72** is their GREATEST COMMON MEASURE.

(7.) *When one number can be measured (i.e., exactly divided,) by two or more numbers, it is said to be a* COMMON MULTIPLE *of these numbers.*

Thus, 72 is a COMMON MULTIPLE of 2, 3, and 4.

(8.) *The least number which is a common multiple of two or more numbers is called their* LEAST COMMON MULTIPLE.

Thus, 144 is a *common multiple* of 8 and 9, but 72 is their LEAST COMMON MULTIPLE.

(9.) The product of two given numbers always furnishes a multiple of these numbers.

This is evident from the diagram given above.

It should now be explained that—

(10.) *The two or more numbers which are multiplied together to form a product, are called its* FACTORS,

From which it follows that—

(11.) *Every product stands to its factors in the relation of a multiple to its measures.*

Thus, 91, which is the product of 7 and 13, is really a *multiple*, of which 7 and 13 are the *measures.*

We may now add that—

(12.) A COMPOSITE NUMBER *is one which is the product of two or more numbers.*

Thus, 28 is a Composite Number because it is the product of 7 and 4.

Hence every *multiple* is a *composite number.*

(13.) A PRIME NUMBER *is one which cannot be resolved into factors—i.e., a number which is not exactly divisible by any number besides itself and unity.*

(14.) *Numbers which have no common measure but unity are said to be* PRIME TO EACH OTHER.*

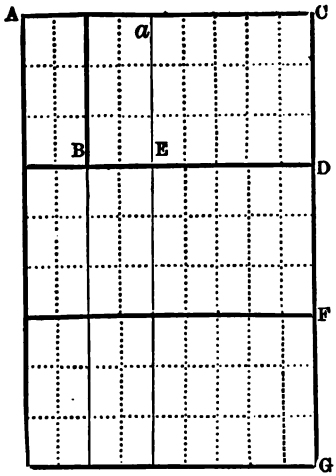
Thus, 6 and 35 are Prime to each other, because neither 2 nor 3, the measures of 6, is a measure of 35.

---

* To test the pupils' apprehension of these definitions, and more firmly to fix them in their minds, the teacher would do well to require the children (1.) to furnish many examples illustrating each definition, (2.) to state each definition in other words, (3.) to learn the above definitions by heart.

48. Of the Rule for finding the Greatest Common Measure—G.C.M.—of two or more given numbers.

Before entering upon the discovery of this Rule, it will be necessary to become acquainted with a proposition which may be apprehended from a study of the accompanying diagram.



Let AB represent SIX, and BC TWENTY-ONE.

It is plain, from the figure, that AB contains 2 **threes**, and BC 7 **threes**; it follows that 3 is a common measure of the given numbers AB and BC.

But AD is the SUM of these given numbers, and the figure shows clearly that this **Sum** is composed of **threes**; it follows that—

(1.) *A Common Measure of any two given numbers is a measure also of their* SUM.

Further, in the figure, *a*B is equal to AB; therefore EC is the DIFFERENCE between AB and BC, and this difference is also composed of **threes**; it follows that—

(2.) *A Common Measure of any two given numbers is a measure also of their* DIFFERENCE.

Again, AF is clearly equal to twice AD, and AG to thrice AD, so that AF and AG are each MULTIPLES of AD—*i.e.*, of the **Sum** of the given numbers AB and BC.

But AF and AG are each composed of **threes**; it follows that—

(3.) *A Common Measure of any two given numbers is a measure also of any* MULTIPLE OF THEIR SUM.

Lastly, *a*F is clearly equal to twice EC, and *a*G to thrice EC, so that *a*F and *a*G are each MULTIPLES of EC—*i.e.*, of the **difference** of the given numbers AB and AC.

But aF and aG are each composed of *threes;* it follows that—

(4.) *A Common Measure of any two given numbers is a measure also of any* MULTIPLE OF THEIR DIFFERENCE.

These results * will be immediately found useful.

Ex. 1. *Let it be required to find the G. C. M. of* 747 *and* 1079.

We know that any number which measures 747 and 1079 must also measure their *difference*—*i.e.*, 332, and also (332 × 2, *i.e.*,) 664, which is *a multiple of this difference.*

Therefore, any number which measures 747 and 1079, must measure also 664.

But, any number which measures 664 and 747, must measure also *their* difference—*i.e.*, 83.

So that any number which measures 747 and 1079, must measure also 664 and 83.

Now, 664 is found, by Division, to be a multiple of 83; therefore 83 measures 664 and itself, and therefore measures also 747 and 1079.

But, 83 is evidently the greatest measure of *itself;* therefore it is the G. C. M. of the given numbers 747 and 1079.

This method of finding the G. C. M. suggests the following

RULE FOR FINDING G. C. M. OF TWO GIVEN NUMBERS.—Divide the greater number by the less; if there be no remainder, the less number is the G. C. M.; but, if there be a remainder, divide the first divisor by this remainder; if there still be a remainder, divide the first remainder by this second remainder. Continue this process till a remainder is found which divides the last remainder exactly. This last divisor is the G. C. M.

As an illustration of this rule, take the above example.

Ex. 1. *Find the G. C. M. of* 747 *and* 1079.

```
747)1079(1
    747
    332)747(2
        664
         83)664(8
            664
```

Therefore 83 is the G. C. M. of 747 and 1079.

Answer, **83.**

---

* The teacher should draw similar diagrams to show that the rule holds good when the Common Measure is 5, 6, 7, &c., and should put such questions as this: "Four measures 12 and 32, therefore it measures——?"

The children answer 44, 20, &c., applying each case of the above rule.

Ex. 2. *Find the G. C. M. of* 372, 558, *and* 961.

1st. We find the G. C. M. of 372 and 558 by the above rule.
This is 186.

2nd. We find the G. C. M. of this 186 and the remaining number 961.
This is 31.

Answer, **31.**

Ex. 3. *Find the G. C. M. of* 1272, 848, 2385, *and* 3975.

1st. We find that the G. C. M. of 1272 and 848 is **424.**
2nd. The G. C. M. of 2385 and 3975 is **795.**
3rd. The G. C. M. of 424 and 795 is **53.**

Answer, **53.**

49. Of the Rule for finding the Least Common Multiple of two or more given numbers.

(1.) From the definition of a *Common Multiple* (§ 46) it is evident that a Common Multiple of two or more given numbers must be a *Composite Number.*

(2.) And since this common multiple is divisible by each of the given numbers, it is evident that *it is exactly divisible by every Prime Factor* contained in the given numbers.*

(3.) Therefore, it may be found by multiplying together

(*a.*) The given numbers ; or,
(*b.*) All the Prime Factors of the given numbers ; or,
(*c.*) One given number by all the Prime Factors of the other given numbers.

Ex. *A Common Multiple of* 15, 42, *and* 66.

(*a.*) $15 \times 42 \times 66$, or,
(*b.*) $(5 \times 3) \times (7 \times 3 \times 2) \times (11 \times 3 \times 2)$ or,
(*c.*) $5 \times 3 \times 7 \times 3 \times 2 \times 66$.

(4.) But, since we have seen (2 above) that the only requisite for a Common Multiple is that it should be exactly divisible by every Prime Factor in the given numbers, it is evident that since 66 is divisible by 2 and by 3 without remainder, it can be made a multiple of **42** by simply

* A *Prime Factor* is a factor which is a Prime Number.

multiplying it by 7—*i.e.*, by the only Prime Factor of 42 of which it is not already a multiple. So that 66 × 7 is a *Common Multiple* of 42 and 66.

It is also the LEAST *Common Multiple* of these two numbers, for it contains no more Prime Factors than are absolutely necessary to constitute it a Common Multiple of them.

Similarly 66 × 7 can be made a multiple of **15**, by multiplying it by 5, the only Prime Factor of 15 of which 66 × 7 is not already a multiple.

From which it follows, as before, that (66 × 7) × 5 is the L. C. M. of the three given numbers.

We thus obtain the following

RULE FOR FINDING THE L. C. M. OF TWO OR MORE NUMBERS.—Strike out of consideration every given number which is contained in any other given number. Of the remaining given numbers, select the highest and multiply it by such Prime Factors of the rest as are not contained in the given number selected.

EX. *Find the L. C. M. of* 14, 110, 42, 220, *and* 63.

1st. Strike out **14** because it is contained in 42, and **110** because it is contained in 220.

2nd. Of the remaining numbers, 42, 220, and 63, select **220**, and multiply it (1.) by 7 and 3, because these are the factors of **42** which are not contained in 220; and (2.) by 3, because the product 220 × 7 × 3 contains but *one* THREE, whereas **63** has *two* THREES among its Prime Factors.

Exercise 22.

Exercises in G. C. M. and L. C. M.

1. Find the G. C. M. of 4455 and 5642; of 721 and 3521; of 2563 and 8701; of 9499 and 2139; of 230 and 7341; of 539 and 336.

2. Find the L. C. M. of 28 and 91; of 95 and 65; of 144 and 169; of 17, 84, and 64; of 35, 95, 164, and 91; of 86, 387, 108, and 70.

3. Of all the numbers, besides Unity, which may be subtracted from 3885 an exact number of times, which is that which may be taken the *greatest*, and which the *least* number of times?

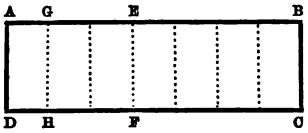
4. Find the G. C. M. of 217, 341, and 310; of 4779, 1475, and 590; of 2268, 1620, and 1377; of 637, 1183, 780, and 715; of 7380, 1968, 2296, 861, and 3895.

CHAPTER VI.

FRACTIONS.

50. Explanations, Definitions, and Introductory Propositions.

Our operations have been hitherto confined to numbers not less than Unity; we shall now proceed to the consideration of numbers both *greater* than, and *less* than, Unity.



(1.) When an apple is cut into 5 equal parts, each part is called *one*-FIFTH; when it is cut into 7 equal parts, each part is called *one*-SEVENTH; and in the accompanying diagram let ABCD represent ONE, or "UNITY." Then each of the seven pieces into which it is divided represents *one-seventh;* and it is clear that *seven-sevenths* make one whole one—*i.e.*, Unity. In the same way it may be shown that—

Unity—i.e., 1 = *six-sixths, nine-ninths, thirteen-thirteenths,** &c.

Again, since AEFD comprises three of these *sevenths*, we say that AEFD is three-sevenths of the unit ABCD; in other words, if ABCD represent ONE, AGHD represents *one*-SEVENTH, and AEFD represents *three*-SEVENTHS.

(2.) Such quantities as *one-seventh, three-sevenths, four-ninths*, &c., are called **Fractions**;† and, in their simplest form, are represented by two numbers of which one is written *above* and the other *below* a line drawn between them; thus—

$\frac{1}{7}$ (*one-seventh;*) $\frac{3}{7}$ (*three-sevenths;*) $\frac{4}{9}$ (*four-ninths*).

An inspection of the above diagram shows that *the number placed below the line* shows into how many parts the unit is divided—in this case SEVEN—and thus gives to these parts their designation—in this case SEVENTHS. It is therefore called the **Denominator** ‡—*i.e.*, the "*namegiver.*"

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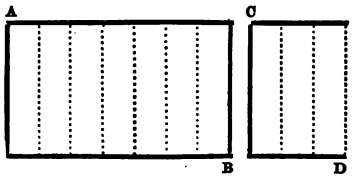
* Let the teacher here put such questions as—"How many *twelfths* make Unity?" "Three whole ones are equal to how many *tenths?*" "A man divided his property equally among his sons; if each received a *fifth*, how many sons had he?" &c.

† From L. *fractus*, broken. ‡ From L. *nomen*, a name.

It is also evident that *the number above the line* shows how many of these parts are under consideration; it is therefore called the **Numerator** *—*i.e*, the "*numberer*" or "*counter*."

(3.) In the accompanying diagram, if AB represent ONE, CD will represent $\frac{3}{7}$ of 1, *i.e.*, $\frac{3}{7}$; and the *sum* of these two figures will be $1 + \frac{3}{7}$, *i.e.*, $1\frac{3}{7}$.



And it is evident that the sum of AB and CD is *Seven-sevenths + three-sevenths, i.e., ten-sevenths, i.e.*, $\frac{10}{7}$.

In this fraction the numerator is greater than the denominator, and such a fraction as this is called an **Improper Fraction**; and such a number as $1\frac{3}{7}$—*i.e.*, a number consisting of integers and fractions, is called a **Mixed Number**.

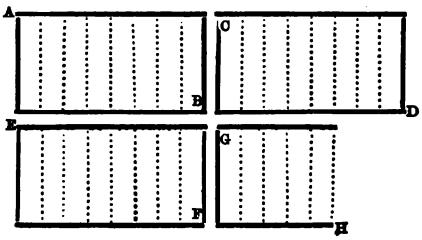
By the aid of the preceding diagram, the following Rule may be understood:—

RULE.—To convert a Mixed Number into an Improper Fraction, multiply the integer by the denominator of the fraction; to this product add the numerator of the fraction. The SUM will form the new numerator, and the new denominator will be the same as the denominator of the given fraction.

Ex. *Reduce* $3\frac{5}{8}$ *to an improper fraction.*

1. BY ILLUSTRATION—

If AB represent ONE divided into *eighths*, then AB, CD, and EF together represent THREE, the sum of whose *eighths* is 3×8, *i.e.*, 24.



But GH also represents $\frac{5}{8}$; therefore the four figures AB, CD, EF, and GH represent $3\frac{5}{8}$, and the total of their eighths is $(3 \times 8) + 5$, *i.e.*, $24 + 5$, *i.e.*, 29; so that $3\frac{5}{8} = 29$ eighths $= \frac{29}{8}$.

* From L. *numerare*, to count.

2. By Reasoning.—

$1 =$ *eight eighths.*

$\therefore 3 = 3$ times *eight eighths* $= 24$ *eighths.*

$\therefore 3 + \frac{5}{8} = 24$ *eighths* $+ 5$ *eighths* $= 29$ *eighths* $= \frac{29}{8}$.

3. By the Rule.—

$$3\tfrac{5}{8} = \frac{(3 \times 8) + 5}{8} = \frac{24 + 5}{8} = \frac{29}{8}, \textit{ Answer.}$$

The converse of this Rule * is also useful, and may be found by an example, thus—

Ex. *Reduce* $\frac{51}{6}$ *to a mixed number.*

Since 6 *sixths* $= 1$, we shall know how may units are contained in $\frac{51}{6}$ as soon as we know how many times 6 is contained in 51.

Therefore we divide 51 by 6, and find that the quotient is 8 and there is a remainder of 3. This 8 evidently signifies 8 *units* and the 3 signifies 3 *sixths.*

Therefore $\frac{51}{6} = 8\frac{3}{6}$.

Hence the following Rule :—

RULE.—To convert an Improper Fraction into a Whole or Mixed Number, divide the numerator by the denominator. If there be no remainder, the quotient, which will be a whole number, will be the answer; but if there be a remainder, the answer will be a Mixed Number, consisting of the quotient as *a whole number,* **and** *a fraction* **having the remainder for its numerator and the original denominator for its denominator.**

(4.) Since the value of an improper fraction may be found by dividing its numerator by its denominator, it would seem that the same rule would apply to all fractions, so that *a fraction might be considered as a simple indication that the number forming its numerator is to be divided by that forming its denominator.*

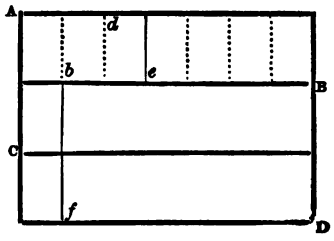
We shall now proceed to show that this is the case.

---

* The teacher should confirm these Rules by setting a great number of such questions as "How many fifths are equal to $1\frac{3}{5}$? How many ninths to $6\frac{4}{9}$?" &c.; and by several such diagrammatic illustrations as the one given above.

Ex. *Let it be required to show that* $\frac{2}{7}$ *of* $1 = \frac{1}{7}$ *of* 3.

1. By Illustration.—



Let AB represent One.

Then AD will represent Three.

Also, Ae will represent $\frac{2}{7}$ of AB, *i.e.*, $\frac{2}{7}$ of 1.

And Af will represent $\frac{1}{7}$ of AD, *i.e.*, $\frac{1}{7}$ of 3.

But the three parallelograms in Af are equal to the three in Ae;

Therefore Ae = Af; *i.e.*,

$\frac{2}{7}$ of $1 = \frac{1}{7}$ of 3.

2. By Reasoning.—

Since 1 = seven-sevenths of a unit,

3 = 21 sevenths of a unit;

$\therefore$ $\frac{1}{7}$ of 3 = $\frac{1}{7}$ of 21 sevenths of a unit = 3 sevenths of a unit

= $\frac{3}{7}$ of 1.

$\therefore$ $\frac{2}{7}$ of $1 = \frac{1}{7}$ of 3.

(5.) It has been already shown that $\frac{4}{9}$ signifies 4 *ninths;* let it now be supposed that we wish to find the value of

5 *times* 4 *ninths.*

We know that 4 *apples* × 5 = 20 *apples*, and 4 *houses* × 5 = 20 *houses;* we therefore argue that 4 *ninths* × 5 = 20 *ninths, i.e.*,

$$\frac{4}{9} \times 5 = \frac{4 \times 5}{9} = \frac{20}{9} = 2\tfrac{2}{9}.$$

Again—*Let it be required to multiply* $\frac{4}{9}$ *by* 3.

Here it is required to make the quotient found by dividing 4 by 9, *three* times greater. This may be done in two ways, viz.—

1st. By making the *dividend* three times *greater*, *i.e.*, by multiplying the 4 by 3;

2nd. By making the *divisor* three times *less*, *i.e.*, by dividing the 9 by 3.

$$\text{Therefore } \frac{4}{9} \times 3 = (1.)\ \frac{4 \times 3}{9};\ \text{and } (2.)\ \frac{4}{9 \div 3}.$$

Hence the following Rule :—

RULE.—To multiply any given fraction by any given integer, multiply the numerator by it, or divide the denominator by it.

(6.) The converse of this Rule may now be found—

Let it be required to divide $\frac{12}{13}$ *by* 4.

The quotient found by dividing 12 by 13 may be made 4 *times less* in two ways, viz.—

1st. By *dividing* its dividend by 4.
2nd. By *multiplying* its divisor by 4.

$$\text{Therefore } \frac{12}{13} \div 4 = (1.)\ \frac{12 \div 4}{13};\ (2.)\ \frac{12}{13 \times 4}.$$

Hence the following Rule:—

RULE.*—To divide a given fraction by any given integer, divide its numerator by it, or multiply its denominator by it.

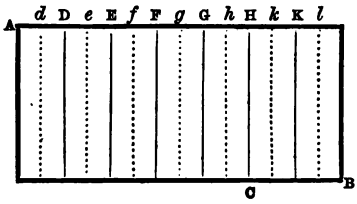
(7.) According to the last Rule it appears that—

$$\frac{12}{13} \div 4 = (1.)\ \frac{12 \div 4}{13},\ i.e.,\ \frac{3}{13};\ \text{and}$$

$$(2.)\ \frac{12}{13 \times 4},\ i.e.,\ \frac{12}{52}.$$

from which it appears that $\frac{3}{13} = \frac{12}{52}$, *i.e.*, $\frac{3 \times 4}{13 \times 4}$, so that we are led to consider that *the value of a fraction is not altered when its numerator and denominator are both* **multiplied** *by the same number;* and that the converse of this proposition is also true—viz., *that the value of a fraction is not altered when its numerator and denominator are both* **divided** *by the same number.*

The truth of this proposition may be shown—



1. By Illustration.—

Let AB represent One divided into Sevenths by the lines D, E, F, G, H, K.

Then AC represents $\frac{4}{7}$.

If each of these *sevenths* be now divided into two equal parts, as by the lines *d*, *e*, *f*, *g*, *h*, *k*, *l*, it is clear that AB

* The teacher should introduce the consideration of such problems as this by such questions as "*Divide* 12 *oranges by* 4. *Divide* 12 *hundreds by* 4." Leading at last to "*Divide* 12 *thirteenths by* 4."

now contains 14 *fourteenths*, and AC 10 *fourteenths—i.e.*, $\frac{10}{14}$. Here we have then—

1. $\frac{5}{7} = \frac{10}{14}$; *i.e.*, $\frac{5}{7} = \frac{5 \times 2}{7 \times 2}$; and, conversely,

2. $\frac{10}{14} = \frac{5}{7}$; *i.e.*, $\frac{10}{14} = \frac{10 \div 2}{14 \div 2}$.

2. By Reasoning.—

If a quantity be first multiplied and then divided by the same number, we know that its ultimate value is the same as its original value.

Now, when the 5 of $\frac{5}{7}$ is multiplied by 2, the value of the $\frac{5}{7}$ is doubled (see par. 5, above), but when the 7 of this fraction is multiplied by 2, its value is reduced one-half (see par. 6, above). So that the value of $\frac{5}{7}$ is not altered by *multiplying* its numerator and denominator by the same number—viz., 2.

The same reasoning applies conversely when the numerator and denominator are both divided by the same number. In this case the process is termed **Cancelling.**

(8.) The preceding proposition leads us to another, viz.—

To reduce a fraction to its lowest terms.

Now, *a fraction is said to be in its lowest terms when its numerator and denominator have no common measure greater than unity.*

The following rule is self-evident—

RULE.—To reduce a fraction to its lowest terms, find the G. C. M. of its numerator and denominator, divide each of them by it, and set down the quotient of the numerator as a new numerator, and the quotient of the denominator as a new denominator.

Ex. *Reduce* $\frac{1533}{2993}$ *to its lowest terms.*

By § 47, we find that the G. C. M. of 1533 and 2993 is 73.

$$\therefore \text{ The answer} = \frac{1533 \div 73}{2993 \div 73} = \frac{21}{41}.$$

(9.) The next proposition which demands attention implies a knowledge of the two last.

To reduce fractions to equivalent ones having a Common Denominator.

Ex. *Let it be required to reduce* $\frac{2}{3}$, $\frac{5}{9}$, *and* $\frac{9}{10}$ *to Equivalent Simple Fractions having a Common Denominator.*

1st. Since $\frac{2}{3}$ is in its *lowest terms*, its numerator and denominator *can-*

not be divided by any number without destroying its character as a Simple Fraction.* It is therefore evident that a simple fraction equivalent to $\frac{2}{3}$ can only be found by multiplying its numerator and denominator by some number, thereby obtaining a denominator which will be a multiple of 3.

2nd. In the same way it may be shown that no simple fraction equivalent to $\frac{5}{9}$ can be found which has not a multiple of 9 for its denominator, nor one equivalent to $\frac{9}{10}$ not having a multiple of 10 for its denominator.†

3rd. We therefore conclude that any common denominator to the three given fractions must be a Common Multiple of their denominators—*i.e.*, must be a Common Multiple of 3, 9, and 10.

But, as small numbers are more easily dealt with than larger ones, the LEAST Common Denominator of such fractions is generally found; and the L. C. Denominator of $\frac{2}{3}$, $\frac{5}{9}$, and $\frac{9}{10}$ is, of course, the L. C. M. of their denominators—*i.e.*, 90 (§ 48).

4th. Taking the fractions in order, we have to multiply 3, the denominator of $\frac{2}{3}$, by 30, to make it 90; and, to keep the value of the fraction unaltered, we multiply 2, its numerator, also by 30,

$$\left.\begin{aligned} \text{and we then have } \frac{2}{3} &= \frac{2 \times 30}{3 \times 30} = \frac{60}{90}, \\ \text{similarly, } \frac{5}{9} &= \frac{5 \times 10}{9 \times 10} = \frac{50}{90}, \\ \text{and } \frac{9}{10} &= \frac{9 \times 9}{10 \times 9} = \frac{81}{90}. \end{aligned}\right\} \text{All these fractions have the same denominator—90.}$$

The following rule is now self-evident—

RULE.—To reduce fractions to equivalent ones having a common denominator, find the L. C. M. of the denominators of the given fractions, and take it for the common denominator. Divide this denominator by each of the given denominators, and multiply each successive quotient by the numerator which corresponds to the denominator used to find that quotient.

---

* *A Simple Fraction* is one whose numerator and denominator are both integers. Such a fraction as $\frac{3}{7\frac{1}{2}}$, or $\frac{4\frac{1}{4}}{9}$, is a *Complex Fraction.*

† In introducing these considerations, the teacher should ask a few questions on *Measures and Multiples*, to recall to the minds of his pupils all they have learnt on these subjects, so that they may not have their attention distracted between trying to remember, for instance, what a multiple is, and endeavouring to follow the demonstration here given. This remark applies universally to cases where knowledge is built upon knowledge.

Exercise 23.

Exercises on Article 50.

1. Reduce to improper fractions :—$1\frac{4}{11}$; $7\frac{3}{8}$; $6\frac{1}{5}$; $101\frac{14}{201}$; $60\frac{85}{707}$.

2. Reduce to mixed numbers :—$\frac{20}{11}$; $\frac{43}{16}$; $\frac{201}{7}$; $\frac{961}{18}$; $\frac{5025}{171}$.

3. Find the least common denominator of the following fractions, and reduce the fractions to equivalent ones having that least common denominator :—$\frac{3}{7}$, $\frac{4}{5}$, $\frac{6}{25}$; $\frac{5}{11}$, $\frac{7}{24}$, $\frac{35}{36}$; $\frac{17}{15}$, $\frac{63}{64}$, $\frac{13}{32}$; $\frac{3}{8}$, $\frac{6}{17}$, $\frac{18}{81}$, $\frac{73}{102}$; $\frac{103}{705}$, $\frac{68}{105}$, $\frac{732}{800}$, $\frac{428}{981}$.

4. Reduce $\frac{13}{40}$, $\frac{18}{35}$, and $\frac{43}{44}$ to equivalent fractions having 24640 as a common denominator.

5. Express $\frac{17}{18}$, $\frac{14}{27}$, $\frac{103}{81}$, $\frac{17}{25}$, $\frac{167}{500}$ as equivalent fractions having 89100 for their common denominator.

6. What is the numerator of a fraction equivalent to $\frac{7}{13}$ if its denominator is 156? and what if its denominator is 533?

7. Find the value of $\frac{7}{9} \times 3$; $\frac{8}{25} \times 7$; $\frac{5}{11} \times 7$; $\frac{16}{45} \times 9$; $\frac{15}{17} \div 3$; $\frac{64}{105} \div 8$; $\frac{104}{323} \div 26$; $\frac{13}{19} \div 8$; $\frac{15}{119} \div 20$.

8. Multiply $\frac{7}{16}$ by 8; $3\frac{1}{25}$ by 7; $5\frac{7}{16}$ by 17; $13\frac{4}{15}$ by 40.

9. Divide $1\frac{9}{16}$ by 5; $8\frac{5}{8}$ by 13; $31\frac{1}{4}$ by 7; $105\frac{16}{45}$ by 71.

10. Reduce to their lowest terms :—$\frac{234}{350}$; $\frac{1333}{1485}$; $\frac{93400}{371448}$.

11. Reduce to whole or mixed numbers :—$\frac{4559}{324}$; $\frac{16065}{2215}$; $\frac{88848}{55555}$; $\frac{460661}{137865}$; $\frac{723509}{18317}$; $\frac{827015}{357455}$.

51. Addition of Fractions

Is performed in four chief cases, viz.—

(1.) *When all the fractions to be added together have the same denominator.*

Ex. 1. *Find the sum of* $\frac{2}{5}$, $\frac{3}{5}$, $\frac{7}{5}$, *and* $\frac{5}{5}$.

Here, 2 horses + 3 horses + 7 horses + 5 horses = 17 horses; and
2 pounds + 3 pounds + 7 pounds + 5 pounds = 17 pounds.

In the same way

2 ninths + 3 ninths + 7 ninths + 5 ninths = 17 ninths—*i.e.*,
$\frac{2}{9} + \frac{3}{9} + \frac{7}{9} + \frac{5}{9} = \frac{17}{9} = 1\frac{8}{9}$, *Answer.*

(2.) *When all the fractions to be added together have* NOT *the same denominator.*

Find the sum of $\frac{3}{5}$, $\frac{4}{7}$, *and* $\frac{31}{40}$.

In Ex. 1, we found the answer by Simple Addition of the given numerators; but this will not give the answer in the present example. For, if we find 38 the sum of the numerators, it is neither 38 *fifths*, 38 *sevenths*, nor 38 *fortieths*.

We therefore, by § 50 (9), express the given fractions as equivalent ones having a common denominator, thus—

$$\frac{168}{280} + \frac{160}{280} + \frac{217}{280}; \left\{ \begin{array}{l} \text{More usually} \\ \text{written thus—} \end{array} \right\} \frac{168 + 160 + 217}{280},$$

and then proceed as in Ex. 1.

The whole process may be shown thus—

$$\frac{3}{5} + \frac{4}{7} + \frac{31}{40} = \frac{168 + 160 + 217}{280} = \frac{545}{280} = \frac{109}{56} = 1\frac{53}{56}, \textit{ Answer.}$$

(3.) *When some, or all, of the fractions to be added together form parts of Mixed Numbers.*

Ex. 3. *Add together* $3\frac{1}{2}$, $2\frac{4}{11}$, $\frac{7}{13}$, and $6\frac{7}{66}$.

The sum of the whole numbers $= 3 + 2 + 6 = 11$.

$$\text{The sum of the fractions} = \frac{1}{2} + \frac{4}{11} + \frac{7}{13} + \frac{7}{66} = \frac{429 + 312 + 462 + 91}{13 \times 66\,*}$$

$$= \frac{1294}{858} = \frac{647}{429} = 1\frac{218}{429}, \textit{ Answer.}$$

(4.) *When some, or all, of the fractions to be added together are improper fractions.*

In such cases the answer may be found as in Ex. 2; but it is preferable to reduce the improper fractions to whole or Mixed Numbers, and then proceed as in Ex. 3.

Ex. 4. *Find the value of* $\frac{22}{3} + \frac{24}{8} + \frac{3}{4}$.

As in Ex. 2:— $\frac{22}{3} + \frac{24}{8} + \frac{3}{4} = \frac{176 + 72 + 18}{24} = \frac{266}{24} = 11\frac{1}{12}$.

As in Ex. 3:— $\frac{22}{3} + \frac{24}{8} + \frac{3}{4} = 7\frac{1}{3} + 3 + \frac{3}{4} = 10 + \frac{1}{3} + \frac{3}{4} = 10 + 1\frac{1}{12} = 11\frac{1}{12}$.

---

* The denominator is kept in this form because it is easier to divide by the given denominators as it now stands—*e.g.*, in the third fraction we have to divide by 13 and then multiply by 7; the division is performed by eliminating the 13 from 13×66.

N.B.—When any of the fractions to be added together are not expressed in their lowest terms, they should first be reduced to their lowest terms and then added together. This remark applies to almost every operation in fractions.

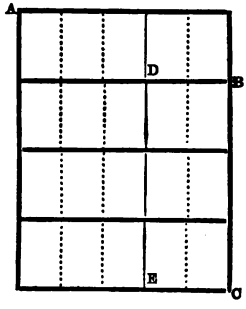
52. Multiplication of Fractions.

The explanations of the following examples will sufficiently illustrate the Rule for Multiplication of Fractions.

(1.) It must first be explained *that* **of** *is used in a sense equivalent to* ×.

Ex. *Let it be required to show that* $\frac{3}{5}$ *of* $4 = \frac{3}{5} \times 4$.

1st. By Illustration.—



Let AB represent One; then AC represents Four.

Also, AD represents $\frac{3}{5}$ of 1; therefore AE represents $\frac{3}{5}$ of 4.

But AE represents also AD × 4; *i.e.*, $\frac{3}{5} \times 4$.

$\therefore$ $\frac{3}{5}$ of $4 = \frac{3}{5} \times 4$.

2nd. By Reasoning.—

$\frac{3}{5}$ of $1 = \frac{3}{5}$.

$\therefore$ $\frac{3}{5}$ of $4 = \frac{3}{5} \times 4$.

(2.) Since $\frac{3}{5} = 3 \div 5$, it follows that 3 is 5 times greater than $\frac{3}{5}$. So that any number being **multiplied** by 3 produces a product 5 times *greater* than the same number would produce if multiplied by $\frac{3}{5}$; and, conversely, any number **divided** by 3 would produce a quotient 5 times *smaller* than the same number would produce if divided by $\frac{3}{5}$.

Ex. 1. *Find the value of* $\frac{3}{10} \times \frac{6}{7}$.

$$\text{Here } \frac{3}{10} \times 6 = \frac{3 \times 6}{10} \text{ (§ 50.—5).}$$

But, according to the explanation immediately preceding, this result is 7 times too great.

$$\therefore \textit{Answer, } \frac{3 \times 6}{10} \div 7 = \frac{3 \times 6}{10 \times 7} \text{ (§ 50.—6).}$$

Hence the following rule :—

RULE.—To find the product of two or more fractions, multiply together all their numerators for a new numerator, and all their denominators for a new denominator.

Ex. 2. *Find the value of* $2\frac{1}{7} \times 7\frac{2}{9}$.

$$\left.\begin{array}{l} 2\frac{1}{7} = \frac{15}{7} \\ 7\frac{2}{9} = \frac{65}{9} \end{array}\right\} = \frac{15}{7} \times \frac{65}{9} = \frac{975}{63} = \frac{325}{21} = \mathbf{15}\frac{10}{21}, \textit{ Answer.}$$

$$\text{Or, by Cancelling, } \frac{\overset{5}{\cancel{15}}}{7} \times \frac{65}{\underset{3}{\cancel{9}}} = \frac{5 \times 65}{7 \times 3} = \frac{325}{21} = \mathbf{15}\frac{10}{21}, \textit{ Answer.}$$

N.B.—1. In finding the value of any such *involved* fraction as the following—

$$6\tfrac{2}{5} \text{ of } \tfrac{7}{16} + \tfrac{4}{9} \times 5\tfrac{1}{16} + 4\tfrac{1}{2},$$

it is necessary to remember that the signs $\times$, $\div$, and *of* connect the terms between which they are found into one quantity, while the signs $+$, $-$, and $\sim$ separate the terms between which they occur into distinct quantities.

So that the given quantity—

$$6\tfrac{2}{5} \text{ of } \tfrac{7}{16} + \tfrac{4}{9} \times 5\tfrac{1}{16} + 4\tfrac{1}{2} = (6\tfrac{2}{5} \text{ of } \tfrac{7}{16}) + (\tfrac{4}{9} \times 5\tfrac{1}{16}) + 4\tfrac{1}{2} =$$

$$\left(\frac{\overset{2}{\cancel{32}}}{5} \times \frac{7}{\cancel{16}}\right) + \left(\frac{\cancel{4}}{\cancel{9}} \times \frac{\overset{9}{\cancel{81}}}{\underset{4}{\cancel{16}}}\right) + 4\tfrac{1}{2} = \tfrac{14}{5} + \tfrac{9}{4} + 4\tfrac{1}{2} = 2\tfrac{4}{5} + 2\tfrac{1}{4} + 4\tfrac{1}{2}$$

$$= \mathbf{9}\tfrac{11}{20}, \textit{ Answer.}$$

N.B.—2. Such a fraction as $\frac{2}{5}$ of $\frac{7}{8}$ is called a **Compound Fraction.**

53. Subtraction of Fractions

Is effected, in the following cases, by methods the reason of which may generally be found by referring to the corresponding cases of Addition of Fractions.

(1.) *When the given fractions have the same denominator.*

Ex. 1. *Find the difference between* $\frac{7}{16}$ *and* $\frac{3}{16}$.

Since $7 - 3 = 4$, it follows that

7 sixteenths − *3 sixteenths* = *4 sixteenths*;

i.e., $\frac{7}{16} - \frac{3}{16} = \frac{4}{16} = \frac{1}{4}$, *Answer.*

(2.) *When the given fractions have not the same denominator.*

Ex. 2. *Find the value of* $\frac{4}{9} - \frac{3}{17}$.

Reducing the given fractions to equivalent ones having a common denominator, we get—

$$\frac{4}{9} - \frac{3}{17} = \frac{68 - 27}{9 \times 17} = \frac{41}{153}, \text{ Answer.}$$

(3.) *When the given fractions form parts of Mixed Numbers.*

The reason of the following Rule is evident:—

RULE.—To find the difference between two Mixed Numbers, find the difference of the whole numbers separately, and also of the fractions separately, and add the two results together.

Ex. 3. *Find the value of* $6\frac{2}{3} - 4\frac{1}{7}$.

$$\left.\begin{array}{l} 6 - 4 = 2. \\ \frac{2}{3} - \frac{4}{7} = \frac{14 - 12}{3 \times 7} = \frac{2}{21} \end{array}\right\} \therefore\ 6\tfrac{2}{3} - 4\tfrac{1}{7} = 2 + \tfrac{2}{21} = 2\tfrac{2}{21}, \text{ Answer.}$$

Ex. 4. *From* $7\frac{1}{3}$ *take* $3\frac{3}{5}$.

$$7 - 3 = 4.$$
$$\frac{1}{3} - \frac{3}{5} = \frac{5 - 9}{15}.$$

Here we want 4 *fifteenths* more to enable us to subtract; we are, as it were, in the position of a debtor who owes 9 *fifteenths* and has only 5 *fifteenths* with which to pay his debt; so that after paying all the *fifteenths* he has, he will still owe 4 *fifteenths*. Now, a debt of 4 *fifteenths* may be appropriately expressed thus, $-\frac{4}{15}$ (*minus* $\frac{4}{15}$).

$$\therefore\ 7\tfrac{1}{3} - 3\tfrac{3}{5} = 4 - \tfrac{4}{15} = 3 + 1 - \tfrac{4}{15} = 3 + \tfrac{15}{15} - \tfrac{4}{15} = 3\tfrac{11}{15}.$$

From this example we learn that *if the fraction in the subtrahend is greater than that in the minuend, the difference of the whole numbers must be* **reduced** *by the difference of the fractions.*

Ex. 5. *From* $19\frac{3}{17}$ *take* $6\frac{5}{8}$.

$$19 - 6 = 13.$$

$$\frac{3}{17} - \frac{5}{8} = \frac{24 - 85}{136} = -\frac{61}{136}.$$

$\therefore$ Answer $= 13$ reduced by $\frac{61}{136} = 13 - \frac{61}{136} = 12 + 1 - \frac{61}{136} =$

$$12 + \frac{136}{136} - \frac{61}{136} = \mathbf{12}\frac{75}{136}, \textit{ Answer.}$$

(4.) *When either the minuend or subtrahend is an Improper Fraction.*

In such cases it is always preferable to reduce the improper fraction to a mixed number, and then proceed as in Exs. 3, 4, 5.

54. Division of Fractions.

It has been shown (§ 52) that if a number be divided by 3, the quotient will be five times less than would be found by dividing that number by $\frac{3}{5}$.

Ex. *Let it now be required to divide* $\frac{7}{8}$ *by* $\frac{3}{5}$.

$$\frac{7}{8} \div 3 = \frac{7}{8 \times 3} \text{ (§ 50.—6).}$$

But this quotient is evidently 5 times smaller than the one required.

$$\therefore \frac{7}{8} \div \frac{3}{5} = \frac{7}{8 \times 3} \times 5 = \frac{7 \times 5}{8 \times 3} \text{ (§ 50.—5).}$$

$$= \frac{7}{8} \times \frac{5}{3} \text{ (By the Rule for Multiplication of Fractions).}$$

Hence the following Rule—

RULE.—To divide one fraction by another, invert the divisor and multiply the fraction forming the original dividend by the inverted fraction.

Ex. 1. *Find the value of* $\frac{3}{7} \div \frac{4}{5}$.

$$\text{By the Rule } \frac{3}{7} \div \frac{4}{5} = \frac{3}{7} \times \frac{5}{4} = \mathbf{\frac{15}{28}}, \textit{ Answer.}$$

Ex. 2. *Divide* $9\frac{3}{8}$ *by* $6\frac{1}{4}$.

$$\text{Here } 9\tfrac{3}{8} \div 6\tfrac{1}{4} = \frac{75}{8} \div \frac{25}{4} = \frac{\overset{3}{\cancel{75}}}{\underset{2}{\cancel{8}}} \times \frac{\cancel{4}}{\cancel{25}} = \frac{3}{2} = \mathbf{1\tfrac{1}{2}}, \textit{ Answer.}$$

Exercise 24.

Exercises in Fractions.

1. Find the value of $\frac{3}{8} + \frac{2}{3} + \frac{4}{5}$; $\frac{13}{15}, \frac{13}{15}, \frac{4}{15}, \frac{13}{15}, \frac{6}{15}, \frac{3}{15}$; $4\frac{1}{7} + \frac{6}{7} + 2\frac{6}{7} + \frac{3}{7}$; $2\frac{5}{12} + 16\frac{11}{12} + 3\frac{7}{12} + \frac{5}{12} + 1\frac{11}{12}$; $2\frac{3}{41} + \frac{18}{41} + \frac{17}{41} + 6\frac{10}{41}$; $17\frac{10}{37} + \frac{19}{37} + 2\frac{16}{37} + 16\frac{36}{37}$; $\frac{108}{43} + \frac{17}{43} + \frac{26}{43}$; $\frac{345}{71} + \frac{109}{71} + \frac{32}{71} + \frac{4}{71} + \frac{16}{71} + \frac{111}{71}$; $\frac{33}{80} + \frac{61}{80} + 14\frac{18}{80} + \frac{640}{80}$; $\frac{500}{99} + \frac{307}{99} + \frac{1080}{99} + 16\frac{47}{99}$.

2. Add together $\frac{1}{3}$, $\frac{2}{7}$, and $\frac{16}{28}$; $\frac{4}{5}$, $\frac{16}{35}$, $\frac{27}{40}$, and $\frac{38}{60}$; $\frac{4}{13}$, $\frac{17}{60}$, $\frac{47}{61}$, $\frac{31}{60}$, and $\frac{21}{52}$; $\frac{73}{95}$, $\frac{103}{105}$, $\frac{4}{7}$, $\frac{13}{56}$, and $\frac{345}{132}$; $4\frac{1}{2}$, $3\frac{1}{4}$, and $7\frac{4}{5}$; $2\frac{3}{11}$, $\frac{14}{27}$, and $6\frac{13}{40}$; $\frac{18}{55}$, $2\frac{13}{21}$, $\frac{71}{80}$, and $8\frac{1}{4}$; $18\frac{471}{1428}$, $301\frac{203}{260}$, $\frac{5}{91}$, $\frac{18}{45}$, and $\frac{4}{9}$ of $\frac{3}{8}$ of $40\frac{1}{3}$.

3. Find the sum of $5\frac{1}{4}$, $\frac{27}{4}$, 8, $\frac{21}{16}$, and $\frac{605}{100}$; of $\frac{13}{36}$, $\frac{107}{48}$, 19, $\frac{1215}{144}$, and $\frac{4}{9}$ of $\frac{5}{8}$ of $7\frac{1}{3}$.

4. Find the difference between $\frac{3}{5}$ and $\frac{3}{8}$; $\frac{7}{13}$ and $\frac{3}{13}$; $\frac{6}{11}$ and $\frac{3}{11}$; $\frac{4}{31}$ and $\frac{13}{31}$; $\frac{29}{36}$ and $\frac{3}{13}$; $\frac{11}{18}$ and $\frac{23}{24}$; $\frac{95}{100}$ and $\frac{5}{20}$; $\frac{71}{101}$ and $\frac{10}{11}$; $\frac{38}{65}$ and $\frac{25}{45}$; $\frac{38}{56}$ and $\frac{35}{60}$; $\frac{91}{147}$ and $\frac{187}{231}$.

5. Find the difference between $3\frac{1}{4}$ and $1\frac{3}{11}$; $16\frac{3}{5}$ and $8\frac{1}{5}$; $10\frac{13}{17}$ and $4\frac{19}{51}$; $2\frac{1}{2}$ and $7\frac{3}{10}$; $14\frac{3}{13}$ and $17\frac{1}{13}$; 2 and $4\frac{1}{7}$; 18 and $13\frac{13}{35}$; $10\frac{101}{111}$ and $3\frac{17}{37}$; $\frac{101}{121}$ and $\frac{17}{33}$; $\frac{317}{1443}$ and $1\frac{131}{231}$; $\frac{409}{595}$ and $\frac{401}{1313}$; $3\frac{193}{504}$ and $3\frac{17}{108}$.

6. What number must be added to $3\frac{1}{3}$ of $\frac{14}{17}$ of 81 to make it equal to $6\frac{3}{17}$ of 81 times $23\frac{1}{3}$?

7. Find the value of $\frac{4}{15} \times \frac{3}{10}$; $\frac{16}{21} \times \frac{39}{56}$; $1\frac{4}{7} \times \frac{34}{121}$; $16\frac{3}{4} \times \frac{3}{14}$; $\frac{3}{7} \times 1\frac{2}{3} \times$ 16 times $3\frac{1}{8}$; $\frac{19}{35}$ of $6\frac{4}{5} \times 7\frac{4}{5}$ of $11\frac{1}{9}$.

8. Find the *product* of the *sum* and *difference* of $\frac{3}{8}$ and $\frac{17}{21}$.

9. What number is that which being divided by $18\frac{3}{7}$ gives a quotient of $3\frac{1}{20}$.

10. By how much must the product of 18 and $9\frac{3}{11}$ be increased to make it equal to $16\frac{1}{3}$ times $\frac{14}{171}$ of $900\frac{1}{2}$?

11. Find the value of $18\frac{1}{4} \div \frac{4}{21}$; $\frac{16}{35} \div \frac{18}{19}$; $3\frac{4}{7} \div 1\frac{2}{3}$; $9\frac{1}{7} \div 3\frac{1}{4}$; $2\frac{3}{8} \div 8\frac{4}{7}$; $17 \div 14\frac{3}{19}$; $\frac{16}{35} \div 3\frac{1}{10}$; $19 \div 40\frac{4}{11}$.

12. What number is that which being multiplied by $18\frac{3}{19}$ gives $8\frac{1}{6}$ for a product?

13. How many times is $65\frac{3}{7}$ contained in $101\frac{1}{2}$?

14. How many lb. of cocoa, at $16\frac{3}{4}$d. per lb., can be purchased for 1 guinea?

15. If the area of a field be $8\frac{3}{10}$ acres, and the land be worth £$6\frac{2}{3}$ per rood, how many acres are contained in a field which is worth £3873, 6s. 8d.?

55. Miscellaneous Examples in Fractions.

(1.) Since the value of a fraction is not altered by dividing its numerator and denominator by the same number, we find that—

$$\frac{4}{5} = \frac{4 \div 3}{5 \div 3} = \frac{1\frac{1}{3}}{1\frac{2}{3}}.$$

Such a fraction as this is called a *Complex fraction.*

(2.) Since $5 \div 1 = 5$, it follows that *any integer may be regarded as a fraction having the integer for its numerator, and unity for its denominator;* thus—

$$8 = \frac{8}{1};\ 7 = \frac{7}{1}, \text{ \&c.}$$

(3.) Let it now be required to find the value of $\frac{7\frac{3}{8}}{6\frac{1}{7}}$.

$$\text{Here } \frac{7\frac{3}{8}}{6\frac{1}{7}} = \frac{\frac{59}{8}}{\frac{43}{7}} = \frac{59}{8} \div \frac{43}{7} = \frac{59}{8} \times \frac{7}{43} = \frac{59 \times 7}{8 \times 43}$$

Hence the following Rule—

RULE.—To reduce a complex fraction to a simple one, express its numerator and denominator as fractions, keeping the given denominator below the given numerator; then multiply together the extremes (*i.e.*, the top and bottom numbers, *in the Ex. above*, 59 *and* 7,) for the numerator of the answer, and the means (*i.e.*, the middle numbers, *in this case* 8 *and* 43) for its denominator.

The process of *cancelling* is thus applied in such cases—

$$\text{Since the } \textit{Answer} = \frac{\text{the product of the extremes}}{\text{the product of the means}},$$

it follows that if either of the *extremes* is divisible by the same number as one of the *means*, we may cancel by this common measure, thus—

$$\frac{8\frac{3}{4}}{5\frac{1}{5}} = \frac{\frac{\cancel{35}^{\,5}}{4}}{\frac{\cancel{26}^{\,4}}{5}} = \frac{5 \times 5}{4 \times 4} = \frac{25}{16} = 1\frac{9}{16}.$$

(4.) Let us now consider a few applications of "Fractions."

Ex. 1. *Find the area of a room which is* $8\frac{1}{2}$ yd. *wide, and* $13\frac{1}{4}$ yd. *long.*

$$\text{Area} = \text{Length} \times \text{Breadth} = 13\tfrac{1}{4}\text{ yd.} \times 8\tfrac{1}{2}\text{ yd.}$$
$$= \left(\frac{53}{4} \times \frac{17}{2}\right)\text{ sq. yd.} = \frac{901}{8}\text{ sq. yd.}$$
$$= \mathbf{112}\tfrac{5}{8}\text{ sq. yd., } \textit{Answer.}$$

Ex. 2. *If a sack of corn weigh* $107\frac{3}{8}$ lb., *what is the weight of corn in* $6\frac{1}{4}$ *sacks?*

$$\textit{Answer} = 107\tfrac{3}{8}\text{ lb.} \times 6\tfrac{1}{4} = \frac{859\text{ lb.}}{8} \times \frac{25}{4} = \frac{21475\text{ lb.}}{32}$$
$$= \mathbf{671}\frac{3}{32}\text{ lb.}$$

Ex. 3. { *Reduce* 4s. 6d. *to the fraction of* 5s. 3d.—
i.e., Express 4s. 6d. *as a fraction of* 5s. 3d.

Here 4s. 6d. = 54d.; and 5s. 3d. = 63d.
And 1d., therefore = $\frac{1}{63}$ of 5s. 3d.
$\therefore$ 54d. = $\frac{54}{63}$ of 5s. 3d.

$$\textit{Answer} = \frac{54}{63} = \frac{6}{7}$$

From this example the following Rule may be deduced:—

RULE.—To reduce a given quantity to the fraction of another given quantity, reduce them both to one denomination, and take the result of the first as the numerator, and that of the second as the denominator, of the required fraction.

Ex. 4. *Which is the greatest, and which the least, of the fractions* $\frac{3}{7}$, $\frac{8}{11}$, *and* $\frac{8}{13}$?

Reducing these fractions to equivalent ones having a common denominator, thus—

$$\frac{429,\ 546,\ 616}{7 \times 11 \times 13},$$

we see that—

$$\frac{3}{7},\ \textit{i.e.},\ \frac{429}{7 \times 11 \times 13},\ \text{is the least, and}$$

$$\frac{8}{13},\ \textit{i.e.},\ \frac{616}{7 \times 11 \times 13},\ \text{is the greatest.}$$

Ex. 5. *Simplify* $\dfrac{42\frac{6}{7} \times 7\frac{4}{5}}{11\frac{7}{10}}$.

$$\left.\begin{array}{r} \text{Here } 42\tfrac{6}{7} = \dfrac{300}{7}\,; \\ 7\tfrac{4}{5} = \dfrac{39}{5}\,; \\ \text{and } 11\tfrac{7}{10} = \dfrac{117}{10}\,; \end{array}\right\} \text{ therefore, } \frac{42\frac{6}{7} \times 7\frac{4}{5}}{11\frac{7}{10}} = \frac{\frac{300}{7} \times \frac{39}{5}}{\frac{117}{10}} = \frac{300}{7} \times \frac{39}{5} \div \frac{117}{10} = \frac{300}{7} \times \frac{39}{5} \times \frac{10}{117}. \text{ (By Rule for Division of Fractions.)}$$

The same result may be obtained by the following method:—

Having reduced the given numbers to simple fractions, take a line and place ABOVE it the *numerators* of those fractions which form the given numerator, and the *denominators* of those fractions which form the given denominator; and BELOW it, place the *denominators* of the fractions whose numerators are above the line, and the *numerators* of the fractions whose denominators are above the line; then simplify the new expression.

This Rule may be illustrated by the above example, thus—

$$\textit{The Answer} = \frac{\overset{100}{\cancel{300}} \times \overset{2}{\cancel{39}} \times \cancel{10}}{7 \times \cancel{5} \times \underset{\cancel{3}}{\cancel{117}}} = \frac{200}{7} = \mathbf{28\tfrac{4}{7}}, \textit{ Answer.}$$

Exercise 25.

Additional Exercises in Fractions.

1. Find the value of $\dfrac{3\frac{3}{7}}{8\frac{2}{3}}$; $\dfrac{7\frac{1}{2}}{3\frac{3}{8}}$; $\dfrac{\frac{4}{5} + \frac{3}{8}}{6\frac{3}{10}}$; $\dfrac{2\frac{4}{5}}{3\frac{16}{25}}$; $\dfrac{17\frac{3}{5} - 2\frac{1}{2}}{4\frac{3}{4} + 3\frac{7}{10}}$.

2. Reduce 3s. 4d. to the fraction of £5.

3. What fraction of £3, 14s. 6d is equal to $\frac{1}{3}$ of 17s. 9d.?

4. What is the value of $\dfrac{2\frac{2}{11}}{8\frac{4}{13}}$ of 431 t. 17 cwt. 3 qr.?

5. Find the value of $\dfrac{17\frac{3}{8}}{4\frac{29}{37}}$ of 360 m. 7 fur. 20 poles, and reduce the result to the fraction of 50 miles.

6. Find the difference between $4\frac{5}{12}$ of £430, 10s. $4\frac{1}{2}$d., and $3\frac{4}{27}$ of £427, 19s. $10\frac{3}{4}$d.

7. Divide 1605 lb. 6 oz. 9 dwt. 19 gr. by $12\frac{1}{7}$.

8. Add together $\frac{4}{15}$ of £83, 19s. 7d. and $6\frac{3}{20}$ of £906, 19s. 2d.

9. What is the difference between $\frac{11}{12}$ of $6\frac{1}{5}$ and $\frac{3}{15}$ of $\frac{4\frac{1}{2}}{7}$?

10. What fraction of £17, 12s. 6d. is $\frac{3}{8}$ of £453, 9s. $4\frac{1}{2}$d.?

11. An army having lost $\frac{3}{13}$ of its men is soon after reinforced with 10800 men, and $\frac{3}{8}$ of its present strength being mustered was found to be 14322 men. What was the original strength of that army?

12. Find the value of $\frac{8\frac{3}{4} \times 19\frac{1}{7}}{2\frac{4}{5}}$; $\frac{46 \times 108\frac{4}{9}}{15\frac{5}{9}}$; $\frac{23\frac{3}{4} \times 19\frac{5}{8} \times 47\frac{5}{11}}{4\frac{3}{17} \times 7\frac{3}{13}}$; $\frac{14\frac{3}{4} \text{ times } 71 \text{ times } \frac{3}{10} \text{ of } 68\frac{4}{9}}{86 \text{ times } 14\frac{3}{4} \text{ times } \frac{3}{11} \text{ of } 400}$.

13. If $\frac{1}{8}$ of the cargo of a ship be worth £305, 19s. 6d., what is the value of $\frac{3}{8}$ of the cargo?

14. If $\frac{3}{13}$ of the value of a horse be £120, 17s. 6d., what is the value of that horse?

15. What is the united area of a field and garden if $\frac{3}{8}$ of the size of the former be 16 ac. 1 r. 35 po., and its whole area be $\frac{7}{13}$ of that of the latter?

16. The combined weight of an engine and tender was 33 tons 17 cwt. The weight of the engine being $\frac{11}{8}$ that of the tender, what was the weight of each?

17. A person having a certain sum of money in the bank, commences to draw out £30, 10s. 6d. yearly, and to deposit £40, 3s. 9d. a year. At the end of 25 years he finds he has raised his money to $3\frac{1}{4}$ of what it was at the beginning of that period. How much had he at first?

18. Divide 1251 tons 3 cwt. into three portions, so that if one portion be represented by $\frac{3}{5}$, the second shall be represented by $\frac{2}{3}$, and the third by $\frac{4}{7}$.

19. A gentleman dies worth a certain sum of money, which is divided among his widow and his two sons. After paying legal and other expenses, the widow's share is paid her, and the eldest son receives a sum of money which was $\frac{5}{11}$ of the total sum left by his father. His brother's share, which was $\frac{4}{15}$ of the share of his mother, was found to be $\frac{4}{37}$ of his father's property after paying expenses. If these expenses were £14, 17s. 1d., what was his father worth at his death, and into what sums was this money divided?

DECIMAL FRACTIONS.

56. Of Vulgar and Decimal Fractions and the Methods of their Mutual Conversion.

(1.) We have hitherto dealt only with such fractions as have their numerators and denominators both expressed—*i.e.*, **Vulgar Fractions.** We shall now proceed to the consideration of another kind of fractions called **Decimal Fractions,** or **Decimals,** which differ from Vulgar Fractions—

1. In never having their denominators *expressed;*
2. In never having for their denominators any number which is not either TEN or some POWER * of TEN.

(2.) Now, in Arts. 7 and 8, it has been shown that *the local value of a digit is increased ten-fold by moving it one place further to the left;* from which it follows that—

The value of a digit is decreased ten-fold by moving it one place further to the right.

Thus, in the number 674 the value of the 7 is 7 *tens*—*i.e.*, 70; in the number 357 the value of the 7 is 7 *ones*—*i.e.*, 7.

Let us now place a *dot* after the units' figure in this last number, and then place after it another 7, thus—

357·7.

Here the 7 on the left of the dot signifies 7 *ones* as before, and analogy seems to persuade us that the 7 on the right of the dot signifies 7 *tenths of one*—*i.e.*, $\frac{7}{10}$.

In the same way it would follow that in the number

368·27,

the 7 would signify 7 *hundredths of one*—*i.e.*, $\frac{7}{100}$.

In this supposition we are right, and we have thus found a method of expressing numbers less than UNITY by means of Simple Notation. This

* A *power* of a number is found by multiplying it by itself a certain number of times. When a number is multiplied by itself *once*, the product is called the *second power* of that number; if the number be multiplied by itself *twice*, the product is called the *third power* of that number, &c. See § 124 of "*Advanced Arithmetic.*"

method is called the **Decimal** * **System of Notation**, because the number TEN forms its basis.

Whenever numbers less than Unity have to be expressed in this Decimal Notation, the *dot* is placed after its units' figure *simply to show which is the units' figure.* Any other plan of indicating this would be equally efficacious but not, perhaps, so expedient; for example—

38·6 might have been expressed thus, $3\underline{8}6$, or 38|6.

This dot is called the **Decimal Point.**

(3.) From what we have said it is clear that numbers *not less than Unity* are expressed by figures placed on the LEFT of the decimal point, while numbers *less than Unity* are expressed by figures placed on its RIGHT.

The whole system of Decimal Notation may therefore be indicated thus—

Thousands.	Hundreds.	Tens.	UNITS.		Tenths.	Hundredths.	Thousandths.
6	3	5	7	·	2	1	9

(4.) We now find no difficulty in expressing such a number as $4\frac{3}{10}$, or $48\frac{37}{100}$, in the Decimal Notation, for—

$4\frac{3}{10} = 4$ *units* and 3 *tenths* = 4·3; and
$48\frac{37}{100} = 48$ *units* and 37 *hundredths* = 48·37 (§ 9, N.B.)

To make this more clear, let it be considered that by § 9, N.B. the number 39·427 may be read in the following manners—

39 *units* 4 *tenths* 2 *hundredths* 7 *thousandths;*
394 *tenths* 2 *hundredths* 7 *thousands;*
3942 *hundredths* 7 *thousandths;*

39427 *thousandths, i.e.,* $\frac{39427}{1000}$.

Similarly ·6193 = 6193 *ten-thousandths* = $\frac{6193}{10000}$.

and ·00385 = 385 *hundred-thousandths* = $\frac{385}{100000}$.

---

* From L. *decem*, ten. Cf. *Decimate*, to take the tenth part of; and *December*, the tenth month—*i.e.*, counting from March.

From which we gather that—

1st. Every finite decimal (§ 56.—2) may be converted into a Vulgar Fraction by taking its significant * figures as the numerator, and the denominator of the last figure on the right as the denominator.

2nd. That this denominator may also be found by writing a cipher for every figure in the given decimal and placing a 1 before it, thus—

$$\cdot 5193 = \frac{5193}{10000}; \quad \cdot 00367 = \frac{367}{100000}.$$

3rd. That any number composed of integers and decimals may be expressed as an Improper Vulgar Fraction having all the figures of the number for its numerator, and the denominator of the last decimal figure for its denominator, thus—

$$43\cdot 018 = \frac{43018}{1000}.$$

(5.) Now, we know that comparatively few fractions have either TEN *or some power of* TEN for their denominators, so that it would appear that only a few Vulgar Fractions can be expressed as Decimal Fractions, unless, indeed, some means can be devised for expressing all, or nearly all, Vulgar Fractions as equivalent ones having *ten*, or some power of *ten*, as their denominators. And here we may remark that *it is not possible to express every Vulgar Fraction as a Decimal, but it is possible to find a Decimal so nearly equivalent to any given Vulgar Fraction as to differ from it by less than any specified quantity.*†

Ex. 1. *Let it be required to express* $\frac{738}{6}$ *in the Decimal Notation.*

By § 50.—4, we find that $\frac{738}{6} = 738 \div 6$.

And, by § 56.—3, the dividend, 738, is a number expressed in the Decimal Notation.

Now, $738 \div 6 = 123$, and this number also is expressed in the Decimal Notation.

So that it would appear that, in some cases at least, *a Vulgar Fraction may be converted into a Decimal by simply dividing its numerator by its denominator.*

---

* *The significant figures* of a decimal fraction are those which, taken by themselves, have some real value—thus, in ·0309, the figures 309 are the significant figures.

† See Arts. 84 and 85 of "*Advanced Arithmetic.*"

Ex. 2. *Let it be required to express* $\frac{147}{5}$ *in the Decimal Notation.*

Here 147 = 147·0, or 147·00, or 147·000, &c.

Therefore $\frac{147}{5} = \frac{147{\cdot}0}{5}$, &c., where it may be remarked that in this case as many ciphers may be placed after the decimal point as we require.

Now, dividing 147 by 5, thus—

$$5 \,\underline{|\, 147}$$
29 and 2 over.

We find that we have 29 *units* in the quotient (which 29 is expressed in the Decimal Notation) and 2 *over*. We therefore proceed to bring down one of the ciphers, thus—

$$5 \,\underline{|\, 147{\cdot}0}$$
$$29{\cdot}4$$

and thus obtain the *Answer* **29·4**, where the dot is placed after the 29 to make that number signify "29 *units*."

So that $\frac{147}{5} = 29{\cdot}4$—*i.e.*, the given Vulgar Fraction, $\frac{147}{5}$, has been converted into a Decimal, 29·4; and this has been effected in the same manner as in Ex. 1—viz., by dividing the given numerator by the given denominator.

Thus we arrive at the following Rule :—

RULE.—To reduce a Vulgar Fraction to an equivalent decimal, after reducing the given fraction to its lowest terms, place a decimal point after the numerator, and divide the numerator by the denominator.

If there should be any "*over*" from this division, annex as many ciphers to the dividend as may be required to find a quotient with no remainder (*i.e.*, none "*over*").

Beneath the decimal point in the dividend place one in the quotient, and this quotient will be the answer.

Ex. 3. *Express* $\frac{31}{4}$, $\frac{209}{8}$, *and* $\frac{616}{20}$ *as decimals.*

$$4 \,\underline{|\, 31{\cdot}00}$$
7·75, *Answer.*

$$8 \,\underline{|\, 209{\cdot}000}$$
26·125, *Answer.*

$$\frac{616}{20} = \frac{616 \div 4}{20 \div 4} = \frac{154}{5} = 5 \,\underline{|\, 154{\cdot}0}$$
30·8, *Answer.*

Ex. 4. *Express* $\frac{3}{4}$, $\frac{7}{64}$, *and* $\frac{31}{800}$ *as decimals.*

Ans., 4 | 3·
·75

64 { 8 | 7·
8 | ·875
·109375, *Ans.*

8 | 31·
100 | ·3875
·003875, *Ans.*

N.B.—In these last examples it may be noticed that instead of annexing the necessary ciphers to the dividend, they are *supposed* to be annexed, and the Division is proceeded with upon this supposition.

57. Of the Elementary Operations with Decimals.

(1.) From what has already been said it is clear that the operation called **Simple Addition** is really performed with numbers expressed in the Decimal Notation—*e.g.*, 26, 307, 9, and 5803 are all numbers expressed in the Decimal Notation, and may therefore not altogether improperly be called *Decimals.* From which it follows that the Rule for Simple Addition will most likely contain a clue to that for **Addition of Decimal Fractions.** Let us see whether this is so.

Ex. *Let it be required to find the sum of* ·6, ·09, *and* ·00124.

1st. By the Rule for Addition of Fractions (§ 51), we find that these fractions must be reduced to equivalent ones having a common denominator, which is easily done by making them each to have the same number of decimal places, thus—

(1.)		(2.)		(3.)	
·6	=	·60000	=	60000	"*Hundred-thousandths.*"
·09	=	·09000	=	9000	
·00124	=	·00124	=	124	
·69124	=	·69124	=	69124	"*Hundred-thousandths*" = ·69124

Here we have three columns, each showing the same result.

In the third column, the exercise is performed as one in Simple Addition of Concrete Numbers.

In the second column, the actual process is the same as in the last, except that the numbers whose sum is found are considered as the numerators of certain fractions having a common denominator (§ 51.—1).

In the first column, we learn that Addition of Decimals is performed in exactly the same manner as Simple Addition, it being in both cases essential that the units of the given numbers should be placed beneath each other, for which purpose the direction is given that in Addition of Decimals, the decimal points in the given numbers are to be placed beneath each other so as to form a vertical column.

(2.) **Subtraction of Decimals** is in the same way performed according to the Rule for Simple Subtraction.

Ex. *Find the sum of* 3·8, 2·006, 381·57; *and the difference between* 70·3 *and* 2·0091.

$$\left.\begin{array}{r} 3\cdot 8 \\ 2\cdot 006 \\ 381\cdot 57 \end{array}\right\} \text{By Simple Addition.} \qquad \left.\begin{array}{r} 70\cdot 3 \\ 2\cdot 0091 \end{array}\right\} \text{By Simple Subtraction.}$$

387·376, *Answer.* **68·2909**, *Answer.*

(3.) **Multiplication of Decimals** is somewhat more difficult.

Ex. 1. *Let it be required to find the value of* 38·937 × 8·2.

By the Rule for Multiplication of Fractions (§ 52) we have—

$$38\cdot 937 \times 8\cdot 2 = \frac{38937}{1000} \times \frac{82}{10} \text{ (§ 55.—4).}$$

$$= \frac{38937 \times 82}{1000 \times 10} \text{ (§ 52.)} = \frac{38937 \times 82}{10000},$$

from which we learn that *the answer may be found by multiplying together the given decimals as though they were integers, and then dividing their product by* 10000.

Now, if any number, say 427, be divided by TEN, the quotient will be one-tenth of that number, so that

$$\left.\begin{array}{l} \text{the 7 } \textit{units} \text{ will become 7 } \textit{tenths}, \\ \text{the 2 } \textit{tens} \text{ will become 2 } \textit{units}, \\ \text{the 4 } \textit{hundreds} \text{ will become 4 } \textit{tens}. \end{array}\right\} \text{So that } 427\cdot \div 10 = 42\cdot 7.$$

This may be expressed by saying that, when a number is divided by 10, the decimal point is removed one place further to the left.

In the same way we may show that

$$427\cdot \div \mathbf{100} = 4\cdot 27; \quad 293 \div \mathbf{1000} = \cdot 293$$
$$38\cdot 5 \div \mathbf{100} = \cdot 385; \quad \cdot 036 \div \mathbf{1000} = \cdot 000036, \text{ \&c.}$$

Therefore the value of $\frac{38937 \times 82}{10000}$ may be found by finding the product of 38937 and 82 and placing a decimal point FOUR places from the right, from which it appears that the *Product must have as many decimal places in it as there are in the multiplier and multiplicand together.*

Therefore the value of 38·937 × 8·2 is found thus—

```
 38·937
    8·2
 ------
  77874
311496
 ------
319·2834, Answer.
```

The *multiplier* contains ONE decimal place and the *multiplicand* THREE; therefore the *product* contains (1 + 3, *i.e.*,) FOUR.

Ex. 2. *Find the value of* ·00712 × ·0000163.

```
      ·00712
    ·0000163
    --------
        2136
       4272
       712
    --------
·000000116056
```

Here the multiplicand contains 5 *places*, and the multiplier 7, therefore the product must contain 12 *places*. But 116056 contains but 6 *figures*, therefore 6 other figures in the shape of ciphers are prefixed, thus giving the *Answer* **·000000116056**.

(4.) **As Division of Decimals** presents some difficulty to beginners, the most *easy*, though not the most *expeditious*, Rule will here be given. Many others have been proposed (of which one is given in § 79 of the "*Advanced Arithmetic*"), and the *reason* of them all is the same as that of the one following, which is only here preferred because it seems to make the least demand upon the mere memory * of the pupils.

Ex. 1. *Find the value of* 334·17 ÷ 42·3.

$$\text{Here } \frac{334{\cdot}17}{42{\cdot}3} = \frac{334{\cdot}17}{42{\cdot}30} = \frac{334{\cdot}17 \times 100}{42{\cdot}30 \times 100} = \frac{33417}{4230} \text{ (§ 57).}$$

Ex. 2. *Find the value of* 192·5 ÷ 3·08.

$$\text{Here } \frac{192{\cdot}5}{3{\cdot}08} = \frac{192{\cdot}50}{3{\cdot}08} = \frac{192{\cdot}50 \times 100}{3{\cdot}08 \times 100} = \frac{19250}{308} \text{ (§ 57).}$$

In these examples we have—

1st. Made the divisor and dividend (*i.e.*, the denominator and the numerator) to have the same number of decimal places, by adding as many ciphers to either of them as are required to make its decimal places equal in number to those in the other.

2nd. We next remove the decimal point from each of them, and then consider these integral numbers as the denominator and numerator of a vulgar fraction.

* It is not here meant that the cultivation of the *memory* is not to be sought, far from it; but our present object is *more especially* to cultivate the pupils' *intellectual power*, and that more particularly in dealing with number.

3rd. And lastly, we reduce this vulgar fraction to a decimal by the rule given in § 55.

These different processes are embodied in the following Rule:—

RULE.—To perform Division of Decimals, make the divisor and dividend to have each the same number of decimal places, and then proceed as in Simple Division till the dividend is exhausted; put now a decimal point after the quotient already obtained, and then suppose the dividend to contain as many more ciphers as are necessary to complete the operation as in the conversion of a vulgar into a decimal fraction.

Ex. 3. *Find the value of* 59·052 ÷ ·703; 11·05 ÷ ·1625; 5·99784 ÷ 8·04; *and* ·0021886 ÷ ·706.

```
·703) 59·052 (84·, Answer.
      5624
      ----
       2812
       2812
       ----
```

Since the divisor and dividend contain each the same number of decimals, there was, in this example, no need for annexing ciphers. Also, there is no need of the decimal point here when the dividend is exhausted, for it so happens that there is no remainder

∴ *Answer* = **84.**

```
·1625) 11·0500 (68· Answer.
       9 750
       -----
       1 3000
       1 3000
       ------
```

The two ciphers on the right of the dividend were annexed to make its decimal places equal in number to those in the divisor. In this case also the decimal point can be omitted from the quotient.

Answer, **68.**

```
8·04000) 5·99784 (·746, Answer.
         5 628000
         --------
           3698400
           3216000
           -------
            4824000
            4824000
            -------
```

The three dark ciphers were annexed to the divisor to make it possess 5 decimal places like the given dividend, 5·99784.

We now had 804000 in the divisor, and 599784 in the dividend. Division being impossible, we put a decimal point in the quotient and placed a cipher on the right of the dividend—(the dark one)—after which the operation proceeded as in the other examples.

```
·7060000) ·0021886000 (·0031, Answer.
          21180000
          706000O
          7060000
```

The four ciphers were annexed to the divisor to make its decimal places equal in number to those in the dividend. But then we had 21886 ÷ 7060000; division was impossible, therefore we put a decimal point in the quotient and annexed a cipher to the dividend. Division still being impossible, a cipher was put in the quotient and another in the dividend. This was repeated, and then the operation proceeded as in former cases.

Exercise 26.

Exercises on Arts. 56 and 57.

1. Express in the Decimal Notation—$\frac{17}{100}$; $\frac{106}{1000}$; $337\frac{4}{10}$; 209 hundreds 2 units 49 hundredths; 16 tens 4 units 2 hundredths 8 thousandths; 305 units 805 thousandths; 31 millions 3 thousands 205 ten-thousandths; 6283 ten-thousands 48 units 15 hundredths 37 ten-thousandths.

2. Convert into Vulgar Fractions or Mixed Numbers—·95; 30·85; 27·061; 1007·015; ·0025; 8·60625.

3. Express as Decimals—$\frac{8}{16}$; $\frac{107}{80}$; $\frac{701}{250}$; $\frac{7188}{1750}$; $1941\frac{1}{50}$.

4. Multiply 316·7 by 10, by 1000, by 100000; and divide ·039816 by 100, by 1000, and by 100000.

5. Add together 20·6, 1007·209, 8·00625, and 10607·8; 1362·53, 8·01996, 23049·5, 400·96, and 52709; 250·7, 83·96, ·000717, ·03473, 61·8, and 29069·7.

6. Find the difference between 807·61 and 93·03816.

7. How much must be added to 403·75 to make it 600·8?

8. What remains when 100 times ·0403 is taken from 10000 times ·008073?

9. By what number does 87·702 fall short of the difference between 30·606 and 100 times 8·05?

10. Find the value of 37·017 × 6·25; 1008·96 × ·375; ·614 × 80·8; ·01519 × 12·5; 2·8 × 17·9 × ·0385; 42·63 × 83000 × 171·7; 304·8 × 601.9 × 81·0365.

11. Multiply 206·018 by 17, and find how many must be added to the product to make it equal to 300 times the sum of 8·013, 25·014, and ·615.

12. The quotient of a division sum being 7839·65, and the divisor being 28·09, what was the dividend?

13. Express in decimals the number of acres in a field which was divided into 73 allotments, each containing ·03875 of an acre.

14. Find the value of 23·5458 ÷ 7·62; 7263·921293 ÷ 30·647; 61·9162 ÷ 8·825; 17·3540814 ÷ ·08368; 13·041567 ÷ 14·637; ·005823875 ÷ ·0625; ·000058092624 ÷ ·06381.

15. What is that number which, being increased by the sum of 6·803 and 105·7, gives a total equal to $\frac{1}{12}$ of 5415·76?

16. What is that number which, being multiplied by 29·08, gives a product greater by 171·525511 than the product of 14·421 and ·709?

17. The French *metre* being 39·3708 English inches, how many metres are equal to 846·4722 English inches?

18. The French *gramme* being 15·432348 English grains, how many English grains are equal to 807·3 grammes?

58. Of Circulating Decimals.

When the denominator of a fraction, which has been reduced to its lowest terms, is found to contain other prime factors than 2 and 5, that fraction cannot be exactly expressed as a decimal.

Ex. 1. *Let it be required to express $\frac{5}{18}$ as a decimal.*

Proceeding by the Rule in § 54—5, we find that the answer is ·2777777, &c.; there is, in fact, no end to the recurrence of the figure SEVEN.

```
18) 50 (·2777
    36
    ---
    140
    126
     ---
     140
     126
      ---
      140
      126
       ---
        14, &c.
```

Ex. 2. *Let it be required to express $\frac{4}{7}$ as a decimal.*

Here also we find that the figures, 571428, reappear, and will continue to repeat themselves as long as we continue to divide.

```
7 | 4·
  ------------------
  ·5714285714 28, &c.
```

From these two examples we learn—

1st. That certain fractions repeat either one figure, or more, when reduced to a decimal form.

2nd. That this repetition commences as soon as a dividend is found similar to a former one—*e.g.*, in Ex. 2 this repetition commences as soon as the remainder 4 is found, this remainder forming with the next cipher a new dividend similar to the first dividend.

It may be added—

3rd. That it is not usual to proceed with the division after a new dividend is found similar to a former one; so that in the working of the above two examples all that part in darker figures may be omitted.

4th. That decimals of which some part recurs again and again, as in the above examples, are called **Recurring** or **Repeating Decimals**; the figures which recur are called the **Period**, or **Repetend**, and are usually indicated by dots placed over the first and last of them; so that

$\cdot 277777$, &c., is represented by $\cdot 2\dot{7}$, and

$\cdot 571428571428$, &c., is represented by $\cdot\dot{5}7142\dot{8}$.

5th. That such a decimal as $\cdot\dot{5}7142\dot{8}$ is called a **Pure Circulator**, because it recurs from the beginning; and that such a decimal as $\cdot 2\dot{7}$ is called a **Mixed Circulator**, because some of its figures recur and some do not.

59. Of the Conversion of Recurring Decimals to Vulgar Fractions.

Ex. 1. *Let it be required to express* $\cdot 5$ *as a Vulgar Fraction.*

Let x = the value of the Vulgar Fraction required;

then $x = \cdot 55555$, &c.,

and $10x = 5\cdot 55555$, &c. (§ 57).

Subtracting downwards we have—

$$9x = 5\cdot 55555, \text{ \&c.} - \cdot 55555, \text{ \&c.} = 5,$$

$$\therefore\ x = \frac{5}{9}.$$

Therefore $\cdot\dot{5} = \frac{5}{9}$. *Answer.*

Ex. 2. *Let it be required to express* $\cdot 2\dot{4}\dot{5}$ *as a Vulgar Fraction.*

Let x = the value of the required Vulgar Fraction;

then $x = \cdot 2454545$, &c.;

therefore $1000x = 245\cdot 454545$, &c. (§ 57—3);

and $10x = 2\cdot 454545$, &c. (§ 57—3).

Subtracting, we get—

$$1000x - 10x = 245\cdot 4545, \text{ \&c.} - 2\cdot 4545, \text{ \&c.},$$

$$i.e.,\ 990x = 245 - 2 = 243;$$

$$\therefore\ x = \frac{243}{990} = \frac{27}{110}.$$

$\therefore\ \cdot 2\dot{4}\dot{5} = \frac{27}{110}$. *Answer.*

From these two examples we obtain the following Rule :—

RULE.—To reduce a Circulating Decimal* to a Vulgar Fraction :— From the number expressed by all the figures of the given decimal subtract the number expressed by those figures which do not recur; the difference so found will be the numerator of the required fraction. For the denominator, take as many nines as there are recurring figures in the given Decimal, and place after them as many ciphers as there are figures in the *fractional part* of the given Decimal which do not recur.

Ex. 3. *Express* $\cdot 31\dot{6}$, $\cdot 4\dot{2}\dot{8}$, $\cdot \dot{1}\dot{4}$, $36\cdot 0\dot{1}\dot{5}$, *and* $8\cdot\dot{7}1\dot{9}$ *as Vulgar Fractions.*

$$\cdot 31\dot{6} = \frac{316 - 31}{900} = \frac{285}{900} = \frac{57}{180} = \mathbf{\frac{19}{60}}, \textit{ Answer.}$$

$$\cdot 4\dot{2}\dot{8} = \frac{428 - 4}{990} = \frac{424}{990} = \mathbf{\frac{212}{495}}, \textit{ Answer.}$$

$$\cdot \dot{1}\dot{4} = \mathbf{\frac{47}{99}}, \textit{ Answer.}$$

$$36\cdot 0\dot{1}\dot{5} = \frac{36015 - 360}{990} = \frac{35655}{990} = \frac{7131}{198} = \mathbf{\frac{2377}{66}}, \textit{ Answer.}$$

$$8\cdot\dot{7}1\dot{9} = \frac{8719 - 8}{999} = \mathbf{\frac{8711}{999}}, \textit{ Answer.}$$

60. Of the Elementary Operations with Circulating Decimals.

Some additional care is necessary in adding, subtracting, multiplying, and dividing decimal fractions when they take the form of Circulating Decimals. The following examples will pretty clearly indicate the *usual* though not the *only* methods by which the required operations with these fractions are performed :—

Ex. 1. *Find the sum of* $4\cdot 9$, $30\cdot 2\dot{3}$, $14\cdot 6\dot{4}\dot{2}$, *and* $\cdot 0\dot{3}\dot{9}$.

$$\begin{aligned} 4\cdot 9 &= 4\cdot 9 \\ 30\cdot 2\dot{3} &= 30\cdot 2333333, \text{ \&c.} \\ 14\cdot 6\dot{4}\dot{2} &= 14\cdot 6422222, \text{ \&c.} \\ \cdot 0\dot{3}\dot{9} &= \cdot 0390390, \text{ \&c.} \\ \hline \therefore \text{ Their SUM} &= 49\cdot 8145945, \text{ \&c.} = \mathbf{49\cdot 81459}, \textit{ Answer.} \end{aligned}$$

* The word *Decimal* is used in this Rule to signify *any number expressed in the Decimal Notation*, even though greater than unity—*e.g.*, 20·712.

Ex. 2. *Find the difference between* $207{\cdot}3\dot{4}$ *and* $18{\cdot}0\dot{6}\dot{7}$.

$$\begin{aligned} 207{\cdot}3\dot{4} &= 207{\cdot}344444, \text{ \&c.} \\ \underline{18{\cdot}0\dot{6}\dot{7}} &= \underline{18{\cdot}067676, \text{ \&c.}} \end{aligned}$$

Their DIFFERENCE $= 189{\cdot}276768$, &c. $=$ $\mathbf{189{\cdot}2\dot{7}\dot{6}}$, *Answer.*

Ex. 3. *Find, correct to six places of decimals, the difference between* $17{\cdot}\dot{0}4\dot{8}$ *and* $301{\cdot}237$.

$$\begin{aligned} 301{\cdot}237 &= 301{\cdot}237 \\ \underline{17{\cdot}\dot{0}4\dot{8}} &= \underline{17{\cdot}048048048, \text{ \&c.}} \end{aligned}$$

Their DIFFERENCE $= 184{\cdot}188951952$, &c. $=$ $\mathbf{184{\cdot}188951}$, *Answer.*

Ex. 4. *Multiply* $7{\cdot}0\dot{3}\dot{6}$ *by* $10{\cdot}2$.

$$\text{Here } 7{\cdot}0\dot{3}\dot{6} = \frac{7036 - 70}{990} = \frac{6966}{990} = \frac{774}{110} = \frac{387}{55},$$

$$\text{And } 10{\cdot}2 = \frac{102}{10} = \frac{51}{5}.$$

$$\therefore\ 7{\cdot}0\dot{3}\dot{6} \times 10{\cdot}2 = \frac{387}{55} \times \frac{51}{5} = \frac{19737}{275} = \mathbf{71{\cdot}77\dot{0}\dot{9}}, \textit{ Answer.}$$

Ex. 5. *Find the value of* $5{\cdot}\dot{1}\dot{7} \times 3{\cdot}208\dot{3}$.

$$5{\cdot}\dot{1}\dot{7} \times 3{\cdot}208\dot{3} = \frac{517 - 5}{99} \times \frac{32083 - 3208}{9000} = \frac{\overset{64}{\cancel{512}}}{\underset{9}{\cancel{99}}} \times \frac{\overset{\overset{7}{\cancel{105}}}{\cancel{2625}}}{\underset{\underset{3}{\cancel{45}}}{\cancel{1125}}} =$$

$$\frac{64 \times 7}{9 \times 3} = \frac{448}{27} = \mathbf{16{\cdot}\dot{5}9\dot{2}}, \textit{ Answer.}$$

Ex. 6. *Find the value of* $6{\cdot}8\dot{5}\dot{4} \div 4{\cdot}1\dot{8}$.

$$6{\cdot}8\dot{5}\dot{4} \div 4{\cdot}1\dot{8} = \frac{6854 - 68}{990} \div \frac{418 - 41}{90} = \frac{6786}{990} \div \frac{377}{90} =$$

$$\frac{\overset{18}{\cancel{6786}}}{\underset{11}{\cancel{990}}} \times \frac{\cancel{90}}{\cancel{377}} = \frac{18}{11} = \mathbf{1{\cdot}\dot{6}\dot{3}}, \textit{ Answer.}$$

From the working of the above examples the pupil may see that, as a general rule, Circulating Decimals are *added and subtracted* after their recurring periods have been sufficiently extended to secure accurate results, while they can only be easily *multiplied and divided* by first reducing them to Equivalent Vulgar Fractions.

Exercise 27.

Exercises in Recurring Decimals.

1. Reduce to vulgar fractions or mixed numbers $2\cdot\dot{7}\dot{9}$; $312\cdot74\dot{8}\dot{0}$; $\cdot\dot{6}17\dot{4}$; $\cdot0008\dot{3}$; $851\cdot0\dot{9}09\dot{1}$; $23\cdot\dot{6}18\dot{4}$; $200\cdot20\dot{0}31\dot{7}$.

2. Reduce to decimals $\frac{17}{33}$; $\frac{61}{82}$; $\frac{53}{90}$; $2\frac{4}{7}$; $15\frac{1}{8}$; $\frac{103}{15}$; $\frac{607}{990}$; $\frac{1869}{1700}$; $\frac{2728}{256}$; $\frac{700}{243}$; $\frac{177}{19}$; $\frac{400}{51}$.

3. Add together $8\cdot53$, $700\cdot00\dot{9}\dot{1}$, $16\cdot\dot{8}\dot{2}$, $300\cdot\dot{1}7\dot{1}$.

4. Find the value of $17 + 38\cdot019 + 46\cdot\dot{4} + \cdot006\dot{3}$.

5. By how many must $16\cdot\dot{0}3\dot{7}$ be increased to make it equal to $300\cdot\dot{4}$?

6. If $39\cdot07$ be added to 3 times $160\cdot\dot{7}\dot{1}$, how many must be taken from the sum to make it equal to 200 times $3\cdot06\dot{4}1\dot{2}$?

7. Find the product of the sum and difference of $20\cdot0\dot{6}$ and $17\cdot00\dot{4}$.

8. Multiply $47\cdot471$ by $300\cdot8$; increase the product by $400\cdot01\dot{7}$; find by how many it must again be increased to make it equal to $800\cdot7 \times 90\cdot4$, and multiply the number so found by $30\cdot\dot{3}$.

9. Find, correct to *three* places of decimals, the value of $900\cdot7 \div \cdot\dot{4}$; $28\cdot\dot{3} \div \cdot0\dot{3}\dot{7}$; $600\cdot016 \div \cdot\dot{0}5\dot{1}$; $\cdot03\dot{8}\dot{4} \div 16\frac{1}{5}$; $209\frac{1}{13} \div 17\cdot\dot{2}0\dot{7}$; $\cdot00\dot{6}\dot{1} \div \cdot00\dot{1}\dot{8}$.

10. Simplify, correct to *seven* places of decimals, $\dfrac{20\frac{1}{4} \times 3}{1\frac{2}{7}}$; $\dfrac{4\frac{4}{5} \times 6\frac{3}{7}}{10\cdot8\dot{3}}$; $\dfrac{\cdot8 \times \cdot091 \times 70\cdot\dot{7}}{106 \div 81\frac{3}{4}}$.

61. Miscellaneous Problems in Decimal Fractions.

Ex. 1. *Reduce* 8s. 3d. *to the decimal of* £1.

By § 54, Ex. 3, we can reduce 8s. 3d. to the *fraction* of £1, thus—

$$\frac{8\text{s. }3\text{d.}}{£1} = \frac{99\text{d.}}{240\text{d.}} = \frac{33}{80}.$$

This *fraction*, $\frac{33}{80}$, may now be reduced to a *decimal*, by § 55, thus—

$$80 \left\{ \begin{array}{r|l} 10 & 33\cdot \\ \hline 8 & 3\cdot3 \end{array} \right.$$

$\cdot4125$, *Answer.**

* For a shorter method, see page 139 of "*Advanced Arithmetic.*"

Ex. 2. *Find the value of* ·625 *of* 1 ton, *and* $\cdot 08\dot{3}$ *of* £27, 10s.

$$\begin{array}{r} \cdot 625 \text{ ton} \\ 20 \\ \hline \dot{1}2\cdot 5\dot{0}\dot{0} \text{ cwt.} \\ 4 \\ \hline 2\cdot\dot{0} \text{ qr.} \\ \hline \end{array}$$

We first multiply by 20 to reduce the *tons* to *cwts.* The fractional part of the *cwts.* is then multiplied by 4 to reduce it to *qrs.*

Answer, **12** cwt. **2** qr.

$$\cdot 08\dot{3} \text{ of } £27, 10s. = \cdot 08\dot{3} \text{ of } £27\tfrac{1}{2} = \frac{5}{6} \text{ of } \frac{£55}{2} = \frac{£55 \times 5}{2 \times 6} = \frac{£275}{12} =$$

£22, 18s. 4d., *Answer.*

Ex. 3. *Reduce* $\frac{31}{42}$ *to a decimal fraction.*

$$42 \left\{ \begin{array}{r|l} 6 & 31\cdot \\ \hline 7 & 5\cdot 1\dot{6}6666\dot{6} \\ \hline & \cdot 7\dot{3}8095\dot{2} \end{array} \right.$$

Answer, $\cdot 7\dot{3}8095\dot{2}$.

A young pupil might suppose that the answer was ·738 because he finds none "*over*" after the 8. He must remember that the 6 is a circulator.*

Exercise 28.

Exercises on Art. 61.

1. Reduce £3, 6s. 8d. to the decimal of £11, and 13 tons 21 lb. to the decimal of 5 tons.

2. Reduce the following sums of money to the decimal of £1—4s. $7\frac{1}{4}$d., £1, 18s. $0\frac{1}{2}$d.; £61, 13s. 2d.; £900, 0s. $10\frac{1}{2}$d.

3. Find the value of ·75 of £103; $2\cdot\dot{6}$ of £100, 17s. 9d.; $4\cdot 8\dot{3}$ of 4 miles 6 fur.; 30·85 of 105 qr. 2 bush.

4. Reduce to decimals $\frac{16}{21}$, $\frac{85}{143}$, $19\frac{20}{63}$, $10\frac{13}{55}$, $6\frac{27}{91}$.

5. Reduce to a decimal $\dfrac{\frac{3}{17} \text{ of } \frac{4}{19} \text{ of } 27}{6\frac{3}{4} \text{ of } \dfrac{2\frac{1}{2}}{104\frac{1}{2}} \text{ of } \dfrac{13}{17}}$.

6. Find the value of x in the following equations:—

$$\frac{7}{12} = \frac{x}{19};\ \frac{3}{16} = \frac{28}{x};\ \frac{20}{31} = \frac{119}{x};\ \frac{x}{71} = \frac{55}{81}.$$

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* See "*Advanced Arithmetic*," page 132.

MISCELLANEOUS EXAMPLES AND EXERCISES.

1. *If a bicycle wheel be* 2 yd. 3 in. *in circumference, how far will it travel in turning* 1020 *times?*

The wheel being 2 yd. 3 in. in circumference, it is clear that at every turn it is propelled just 2 yd. 3 in.

Therefore 1020 turns propels it 2 yd. 3 in. × 1020.

The answer may now be found in several ways—

1st. **By Multiplication**—According to Rule in § 42.

2nd. **By Fractions**—

$$2 \text{ yd. } 3 \text{ in.} \times 1020 = 2\tfrac{1}{12} \text{ yd.} \times 1020 = \frac{25}{\underset{1}{\cancel{12}}} \text{ yd.} \times \frac{\overset{85}{\cancel{1020}}}{1} = 25 \text{ yd.} \times 85 =$$

2125 yd., *Answer.*

3rd. **By Fractions and Reasoning**—

If the circumference were 1 yd., then, in 1020 turns the wheel would travel 1020 yd. Therefore we get

The distance travelled with a circumference of 1 yd. = 1020 yd.

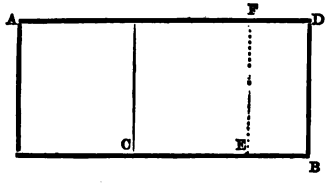
∴ When the circumference is 2 yd. the distance will be 2040 yd.

And when the circumference is $\frac{1}{12}$ yd. the distance will be $\frac{1}{12}$ of 1020 yd., *i.e.* . . . . 85 yd.

∴ When the circumference is 2 yd. + $\frac{1}{12}$ yd. the distance will be 2040 yd. + 85 yd. **2125** yd.

Answer, **2125** yd.

2. *Divide* 40 acres *of land between two brothers, giving the elder one* 7 ac. 2 r. *more than the younger.*



Let AB represent the whole 40 acres,* AC the younger brother's share, and CD the elder brother's share.

Then it is clear that CD is greater than AC by 7 ac. 2 r. Therefore let ED represent this 7 ac. 2 r.

* In teaching arithmetic to beginners, it is always advisable to use diagrams and other illustrations as freely as possible. These aids should afterwards be dispensed with, and the child be required to grasp the problem without their assistance.

It follows that FC is equal to AC, and that

AE = (1.) 40 ac. − 7 ac. 2 r. = 32 ac. 2 r.

(2.) Twice the share of the younger brother.

∴ The share of the younger brother =

32 ac. 2 r. ÷ 2 = **16** ac. **1** r.

And that of the elder brother =

16 ac. 1 r. + 7 ac. 2 r. = **23** ac. **3** r.

Answer.

3. *If* $\frac{8}{27}$ *of the value of a ship be* £700, 15s., *what is the value of* 3 *such ships?*

$\frac{8}{27}$ of a ship is worth £$700\frac{3}{4}$;

∴ $\frac{1}{27}$ of a ship is worth $\frac{£700\frac{3}{4}}{8}$;

∴ All the ship is worth $\frac{£700\frac{3}{4}}{8} \times 27 = \frac{£700\frac{3}{4} \times 27}{8}$ (§ 50—5);

∴ 3 such ships are worth $\frac{£700\frac{3}{4} \times 27 \times 3}{8} = £\frac{2803 \times 27 \times 3}{4 \times 8} = £\frac{227043}{32}$

= **£7095, 1s. 10½d.**, *Answer.*

4. *Find the value of* $8\frac{4}{5}$ *tons of metal at* £69, 10s. 10d. *per ton.*

The value of 1 ton = £ 69, 10s. 10d.

8

∴ The value of 8 tons = £556, 6s. 8d.

And the value of $\frac{1}{5}$ of a ton = $\frac{£69, 10s. 10d.}{5}$ = £ 13, 18s. 2d.

∴ the value of $\frac{4}{5}$ of a ton = £13, 18s. 2d. × 2 = £ 27, 16s. 4d.

∴ the value of $8\frac{4}{5}$ tons = **£598, 1s. 2d.** *Ans.*

5. *What sum of money is the highest common measure of* £17, 5s. 2d. *and* £35, 18s. 10d.? (National Society's Middle-Class Examination.)

There are two methods of doing this—

1st. *By reducing each given quantity to pence,* thus—

£17, 5s. 2d. = 4142d.
£35, 18s. 10d. = 6226d. } The G. C. M. of 4142 and 6226 is 38.

Therefore, *Answer* = **38**d. = **3s. 2d.**

2nd. *Without reducing either of the given quantities*, thus—

£17, 5s. 2d.) £35, 18s. 10d. (2
£34, 10s. 4d.

£1, 8s. 6d.) £17, 5s. 2d. (12
£17, 2s. 0d.

3s. 2d.) £1, 8s. 6d. (9
£1, 8s. 6d.

Answer, **3s. 2d.**

6. *If* 3 *ducks are worth* 4 *chickens, and* 3 *geese are worth* 10 *ducks, find the value of a goose; a pair of chickens being worth* 4s. 6d. (Cambridge Local Examinations.)

The value of 3 ducks = that of 4 chickens—*i.e.*, two pair of chickens
= 4s. 6d. × 2 = 9s.

∴ The value of 1 duck $= \frac{9s.}{3} = 3s.$

∴ The value of 10 ducks = 3s. × 10 = 30s.
= also, that of 3 geese.

∴ The value of 3 geese = 30s.

∴ The value of 1 goose = **10s.**, *Answer.*

7. *A person buys a lot of old timber for* £16, *and forthwith sells it for* 20 *guineas. What does he gain on every shilling that the timber cost him?* (National Society's Middle-Class Examination.)

His total gain = 20 guineas − £16 = £5 = 1200d.

∴ The sum gained on [£16, *i.e.*], 320s. = 1200d.

∴ The sum gained on 1s. $= \frac{1200d.}{320} =$ **3¾d.**, *Answer.*

8. *Four-seventeenths of half the seventh part of a gold coin have been worn away, and it now weighs* 5 dwt. $1\frac{5}{28}$ gr. *Find its original weight.* (National Society's Middle-Class Examination.)

$$\text{Here } \frac{4}{17} \text{ of } \frac{1}{2} \text{ of } \frac{1}{7} = \frac{\overset{2}{\cancel{4}}}{17} \times \frac{1}{\underset{\cancel{2}}{2}} \times \frac{1}{7} = \frac{2}{119}.$$

Therefore, we find that—

$$\frac{2}{119} \text{ of the coin have been worn away, and that}$$

$$\therefore \frac{117}{119} \textit{ of the coin weigh } 5 \text{ dwt. } 1\tfrac{5}{28} \text{ gr.} = \frac{3393}{28} \text{ gr.}$$

$$\therefore \frac{1}{119} \textit{ of the coin weighs } \frac{3393 \text{ gr.}}{28} \div 117 = \frac{3393 \text{ gr.}}{28 \times 117}.$$

$$\therefore \textit{The coin weighed } \frac{3393 \text{ gr.}}{28 \times 117} \times 119 = \frac{\overset{29}{\cancel{3393}} \text{ gr.} \times \overset{17}{\cancel{119}}}{\underset{4}{\cancel{28}} \times \underset{1}{\cancel{117}}} = \frac{493 \text{ gr.}}{4} =$$

$$5 \text{ dwt. } 2\tfrac{3}{4} \text{ gr.} = \frac{493 \text{ gr.}}{4} = 123\tfrac{1}{4} \text{ gr.} = 5 \text{ dwt. } 3\tfrac{1}{4} \text{ gr., } \textit{Answer.}$$

Exercise 29.

Miscellaneous Examples.

1. Find the value of £24, 17s. $1\frac{1}{2}$d. × 25.

2. Find the product of 8407 and 90036 and divide it by 21.

3. How many times will a cart wheel turn in 15 miles if it be 13 ft. 4 in. in circumference?

4. How many times may 85 be taken from 130614, and how many will remain?

5. If a coal waggon contains 3 tons 5 cwt. of coals, how many waggons will be required to carry 334 tons 15 cwt.?

6. If $\frac{3}{8}$ of a piece of land be worth £800, 16s. 9d., what is the value of the whole piece?

7. What was the price of an acre of land when $4\frac{1}{3}$ acres cost £132, 5s. 6d.?

8. What is that sum of money of which $\frac{4}{11}$ is £81, 13s. 6d.?

9. What is the price of 7 horses if £200 be the price of 3?

10. What is the circumference of a bicycle wheel which turns 100 times in 250 yards?

11. A boy once subtracted 8017 from a number instead of adding it. If his answer came to 26049, what would the true answer have been?

12. What fraction of £50 is £2, 17s.?

13. How many sq. yd. of turf will be required for a lawn which is $73\frac{1}{3}$ yd. long and 69 yd. wide? There is a gravel walk round it 2 yd. wide; what will be the cost of putting down new gravel at $7\frac{1}{2}$d. per sq. yd.?

14. Simplify $\frac{36 \times 4\frac{2}{7}}{6\frac{1}{4}}$; $\frac{30\frac{2}{13} \times 6\frac{2}{7}}{19\frac{3}{4}}$; $\frac{17\frac{1}{3} \times 10\frac{2}{11}}{53\frac{1}{3}}$.

15. If $\frac{4}{15}$ of a sum of money be £16, 10s., how much is $\frac{18}{35}$ of it.

16. How many quarter of a pound packets of tea can be made up from $13\frac{1}{2}$ lb.?

17. If a hoop be 1 yd. 3 in. round, how far must a boy run with it to turn it 1300 times?

18. How far will a man travel by taking 519 steps, if he takes a full pace of 31 inches?

19. What is the cost of painting a house at 1s. 6d. per sq. ft., if there are altogether 153 sq. yd. to be painted?

20. For what sum was an estate sold if $\frac{4}{9}$ of the purchase money be £7023, 16s. 10d.?

21. What number is that which, being divided by $1\frac{23}{67}$, produces a quotient equal to $\frac{134 \times 55 \times 67}{17 \times 6 \times 16\frac{2}{17}}$?

22. What is the value of the contents of a till which contains 5 sovereigns, 7 half-sovereigns, a crown piece, 19 half-crowns and as many florins, 71 sixpences, and £1, 16s. $4\frac{1}{2}$d. in other money.

23. The value of a bag of money which contained 10 sovereigns, 18 half-sovereigns, £21, 3s. $4\frac{1}{2}$d. in small money, and 41 *other pieces of money*, was £44, 5s. $4\frac{1}{2}$d.; what were those *other pieces of money*?

24. The circumference of a circle being 3·1416 times its diameter, find how long the point of the minute hand of a church clock would be travelling 5066·77248 yd., the hand being 4·2 yd. long.

25. If 49 acres of land be bought for £301·$\dot{1}4285\dot{7}$, how much land can be bought for £38·$\dot{2}\dot{7}$? [By Vulgar Fractions.]

26. The weight of 14·3 bushels of potatoes being ·25 of 10 cwt., what is the weight of 51 bushels?

27. After a number had been reduced by 605, it was divided by 8009, and the quotient was 73018. What was the original number?

28. What was the circumference of a bicycle wheel which turns 2065 times in a mile and a half?

29. If a man earns 6s. 5½d. per day, how long will it take 7 men to earn £770, 16s. 0½d.?

30. How many horses, each worth 33 guineas, are equal in value to 48 acres of land at £187, 13s. 9d. per acre?

31. A woman sold a certain number of fowls at 7s. 6d. a couple; if she received £19, 2s. 6d. for the lot, how many fowls did she sell?

32. If $\cdot\dot{2}0987654\dot{3}$ of a share in a silver mine be worth £358, 18s. 3d., what is the value of 500 shares?

33. If a ton of coals be worth £1, 13s. 6d., what is the value of $7\cdot\dot{3}$ tons?

34. The length of ·6 of a field being 2 poles 4 yd., what is its entire length?

35. What is the value of $\frac{8}{15}$ of an acre of land at £130, 10s. 11d. per acre?

36. Find the value of $3\frac{1}{15}$ of £13334, 11s. 6½d.

37. The circumference of a hoop being 19·03 ft., how long will a boy be travelling 2·5 miles with it, supposing him to turn his hoop 15 times a minute?

38. Find a number which, being multiplied by 307 and then reduced by 1899, produces the product of 605 and 18017.

39. A man made 1831 pins a day; after working a certain number of days he found that as soon as he had made 506 more he would have altogether 136000 pins; how many days had he been at work when he discovered this?

40. What is that number which, being added to the sum of 209·036, 7·8, $40\cdot339\dot{3}$ and $7\cdot80\dot{5}$, produces a number equal to the product of 301·7 and 28·3?

41. If 1·375 of a field be 34 ac. 2 r. 24 poles, how many sq. yd. would be contained in a field which was 6·3 times as large?

42. Simplify $\dfrac{\cdot8 \text{ of } \frac{8}{7} - \cdot\dot{8} \text{ of } \cdot\dot{9}\dot{0} + \cdot\dot{6}}{\frac{4}{7} \text{ of } 1\cdot125 + \frac{2}{3} \text{ of } \cdot5} \times \dfrac{\frac{8}{21} \text{ of } 1\cdot375}{\frac{4}{17} \text{ of } 1\cdot\dot{8} - \cdot\dot{7} \text{ of } \cdot\dot{2}\dot{7}}$.

43. Add together $\cdot3\dot{5}$ of 10s.; $\cdot54\dot{1}8\dot{9}$ of 12s. 4d.; and $\cdot23\dot{8}$ of 12s. 6d.; and reduce the result to the decimal of £1. (*Christ's Hospital. Midsummer* 1872.)

44. The weight of a cubic foot of sea water is 2 qr. 8 lb. 8¼ oz.; a cask floating therein displaces 5 cub. ft. 937½ cub. in. of water. Find the weight of the cask, or, which is the same thing, the weight of water displaced. (*Christ's Hospital. Midsummer* 1872.)

45. If a bar of silver weighing 4·5 lb. be worth £13, 16s. 9d., how many ounces of it are worth £2, 9s. 6½d.?

[*N.B.*—This and the following *ten* exercises are taken from the National Society's Examination of Middle-Class Schools.]

46. Add $17\frac{5}{12}$, $18\frac{7}{18}$, and $19\frac{23}{24}$, and multiply their sum by $\frac{23}{72}$.

47. From $2080\frac{3}{10}$ subtract $1090\frac{3}{15}$, and divide the remainder by $107\frac{7}{12}$.

48. In what number is 41 contained 271 times?

49. How many years of 365¼ days are there in 75972 days?

50. If 52 men can excavate 1250 cub. yd. of clay in 24 hours, in what time could 39 men do the same?

51. If 15 oz. 12 dwt. 13 gr. of silver be worth 80s. 8d., what quantity is worth 50s. 5d.?

52. Mr James Watts owes Mr Thomas Jones for 4 guineas worth of silk at 5s. 3d. per yard, and for 8s. 9d. worth of linen at 2s. 4d. per yard. Make out the bill specifying the quantities.

53. Divide £247 by 5s. 4d., and 247 by 5⅓.

54. Each side of a square surface measures $4\frac{29}{55}$ ft. What is the area?

55. Find the value of $\frac{13}{27} + \frac{17}{18} - 1\frac{5}{12}$; $\dfrac{1\frac{3}{8} + 2\frac{5}{11} + 3\frac{7}{13}}{3\frac{5}{6}}$; 100·01 ÷ ·548; $\dfrac{\cdot 3753}{100\cdot 08}$; $\dfrac{1 \text{ cwt. } 12 \text{ oz.}}{1 \text{ cwt. } 3 \text{ qr. } 9 \text{ lb.}}$ reduced to a decimal.

56. A piece of land is worth £10 per rood; if a field contains 28 ac. 3 r., what is the value of $\frac{4}{5}$ of that field?

57. How much beer is contained in 38 casks which are full and another which is $\frac{3}{4}$ empty, if it is found that a cask which is $\frac{19}{27}$ full contains 48 gallons?

58. In a cricket match the average score on the winning side was 13; the total number of runs on the losing side was $\frac{1}{3\cdot 25}$ of those on the winning side. What were the two scores, eleven playing for each side?

59. A house and garden occupy 3 ac. 2 roods of ground; the garden is five times larger than the house; what is the area of the garden?

60. The first of two trucks of coal contained $1\frac{3}{8}$ of that in the second. If the first contained 19 tons 4 cwt. 97 lb., how much did the second contain?

61. One-third of a pipe of wine being worth £13, 2s. 10d., what is the value of three-elevenths of a pipe?

62. What is the cost of telegraphing to a distance of 3800 miles a message consisting of 300 English and 150 Persian words, at the rate of a $\frac{1}{4}$d. per mile for every 50 English and $\frac{3}{4}$d. per mile for every 30 Persian words?

[*N.B.*—This and the following *five* exercises are from the Cambridge University Local Examinations.]

63. Fifty members pay $7\frac{1}{2}$d. each per week for 20 weeks into a coal club. To this are added five subscriptions of a guinea each, and ten of 10s. 6d. each from other sources. The cost of keeping the accounts being 10s., how much coal at £1, 13s. per ton can be provided out of the remaining funds for each of the fifty members?

64. Find the value of $\frac{1}{2} + \frac{3}{8} + \frac{4}{15} + \frac{7}{24}$ and $\dfrac{3\frac{1}{2} - 1\frac{7}{8}}{3\frac{1}{4} \times 2\frac{1}{2}}$.

65. The water of a certain company contains ·002 of organic impurity, ·003 of metallic impurity, and ·02 of chalk. Find what vulgar fraction of the whole is pure water.

66. A cubic foot of wood weighs 20 lb.; find the weight of 10 planks, each 30 ft. long, 1 ft. wide, and 1 inch thick.

67. Find the value of $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$; $(4\frac{3}{5} + \frac{7}{8})$ of $(\frac{3}{7} + 1)$; $\dfrac{210720 \cdot 6}{\cdot 4206}$; $\dfrac{£1,\ 16s.\ 11d.}{£33,\ 4s.\ 6d.}$ reduced to a decimal.

68. Find the cost of $39\frac{3}{11}$ tons of raisins at £17, 8s. 6d. for half a ton.

69. If $\frac{5}{12}$ of a piece of land contain 18 ac. 3 r. 20 poles, how many sq. yd. are contained in $\frac{7}{9}$ of that piece of land?

70. Find the value of $35\frac{5}{7}$ of £2133, 13s. $10\frac{3}{4}$d.

71. If a mile of telegraph wire costs £3, 14s. 5d., find the cost of $76\frac{7}{15}$ miles.

72. Multiply 5 m. 3 fur. 20 p. 2 yd. by $17\frac{4}{11}$.

Examination at Merchant Taylors' School (March 1874).

73. Reduce 31624516 ounces to tons.

74. How many persons can receive £3, 8s. $6\frac{1}{2}$d. out of £448, 18s. $11\frac{1}{2}$d.?

75. If 113 books cost £20, 7s. $4\frac{1}{2}$d., how many can be bought for £63, 12s. $4\frac{1}{2}$d.?

76. Find the value of

$$\frac{256{\cdot}3424 \times 50{\cdot}3}{{\cdot}032}\,;\ \frac{£18,\ 7s.\ 1\tfrac{1}{2}d.}{£31,\ 8s.\ 6\tfrac{1}{2}d.}\,;\ \frac{46\ lb.\ 1\ oz.\ 10\ dwt.\ 1\ gr.}{5\ oz.\ 3\ dwt.\ 11\ gr.}$$

77. What is the difference in the cost of 432 things at $3\frac{1}{2}$d. each, and 93 things at 1s. $4\frac{1}{4}$d. each?

78. Reduce 47 ac. 3 r. 14 perches to the fraction of 1 acre, and multiply £180, 10s. 6d. by the result.

79. Reduce 15 cwt. 2 qr. 11 lb. to the decimal of 1 cwt., and multiply £6, 15s. 4d. by the result.

80. In a scramble of a certain sum of money, it was ascertained whither $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of it had gone, but the remainder, two shillings, was missing; how much money was scrambled?

81. A pays £91, 5s. a year for his house, and B pays £109, 10s. for his house; if A lends his house to B for 210 days, and B lends his house to A for 175 days, which will be indebted to the other?

82. The sum of £325, 2s. $2\frac{1}{2}$d. is to be divided equally amongst 112 persons, with the exception of the one who divides it, and he is to receive £2, 17s. $6\frac{1}{2}$d. extra for his trouble; how much does each of the others get?

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83. Reduce $\frac{14}{22}$ to a decimal, and say what recurring decimal cannot be found by the ordinary process of Division.

84. Find the cost of 8·75 cwt. of salt at £2·81$\dot{6}$ per cwt.

85. The cost of 1 ton of copper ore being £47·475, find the cost of 95·0$\dot{3}$8095$\dot{2}$ of a ton.

86. If 1 lb. of gold be worth £38, 10s. 9d., what is the value of $7\frac{5}{12}$ lb. of a metal which is only half as valuable as gold?

87. Two casks of brandy cost £36·8$\dot{3}$; what is the value of the brandy in 57 full casks and another which is only $\frac{17}{18}$ full?

88. If $\frac{3}{8}$ of the cargo of a ship be worth £780, 10s., what is the value of $\frac{5}{12}$ of the cargo?

ANSWERS.

Exercise 1.

62; 37; 41; 38; 85; 74; 83; 55.

Exercise 2.

1. Three hundred and twenty-seven; four hundred and sixty-two; eight hundred and ninety-one; seven hundred and twenty-three; five hundred and thirty-six; nine hundred and fifty-five; four hundred and thirty-eight; six hundred and forty-nine; eight hundred and eighty-eight; nine hundred and ninety-nine.

2. 486; 365; 578; 931; 262; 129; 757.

Exercise 3.

1. Twenty-seven thousand four hundred and twenty-six; thirty-eight thousand two hundred and nineteen; five hundred and thirty-two thousand seven hundred and forty-six; two hundred and eighty-one thousand one hundred and seventy-nine; fifty-eight thousand six hundred and twenty-three; seventy-three thousand two hundred and eighty-five; two million three hundred and forty-one thousand two hundred and fifty-two; five million seven hundred and twenty-six thousand eight hundred and thirty-nine; sixty-three million two hundred and forty-seven thousand eight hundred and twenty-six; three hundred and ninety-one million two hundred and thirty-three thousand four hundred and seventy-four; eighty-five million six hundred and ninety-seven thousand three hundred and thirty-three.

2. 5462; 7831; 87452; 6651; 941624; 44899; 2555366; 583771222; 888777333.

Exercise 4.

1. Seventeen; twenty-six; eighteen; eighty; four hundred and three; one thousand seven hundred and nine; sixty-three thousand and eighty; fifty-four thousand six hundred; three thousand and ninety; seven thousand nine hundred and two; two hundred and eighty thousand six hundred and one; seven hundred thousand and sixty-one; two million eighty thousand six hundred and thirty-nine; fifty million two hundred thousand seven hundred and one; one hundred and eighty million six hundred and one thousand and seventeen; five hundred million one thousand and six; three hundred and one million and ninety; eight hundred and twenty million sixteen thousand seven hundred.

2. 19; 70; 605; 3802; 90650; 17100; 200002; 5016101; 26000019; 100002600.

Exercise 5.

1. 20; 15; 17; 28; 55; 57; 66; 160; 210; 610; 300; 40; 19; 99; 5006; 151562.

2. XXX; LXX; XII; XV; XVIII; XXVII; XXXVI; LI; CVIII; LXIII; CCXX; $\overline{\text{IV}}$XVII; $\overline{\text{LXIII}}$DCCCC; $\overline{\text{CLXX}}$CVIII; $\overline{\text{LVI}}$CIII; $\overline{\text{CLXXXIV}}$LCCCCVII.

Exercise 6.

1. 166.
2. 154.
3. 1625.
4. 1584.
5. 29096.
6. 23701.
7. 163465.
8. 2211844.
9. 22009.
10. 170889.
11. 531146.
12. 1827197.
13. 3697034.
14. 134254008.

Exercise 7.

1. 1s. 9d.
   2s. 8d.
   9s. 1d.
   £1, 12s. 7d.
   3s. 4½d.
   £1, 7s. 11¼d.
   £32, 14s. 1d.
   £30, 1s. 9d.
   £349, 15s. 2d.
2. £8, 1s. 7¾d.
3. £70, 19s. 6d.
4. £511, 17s.
5. £110, 13s. 4d.
   £181, 6s. 8d.
6. £5926, 16s. 5d.

Exercise 8.

1. 9732.
2. 25524.
3. 136565.
4. 428988.
5. 550962.
6. 305928.
7. 3812562.
8. 80020809.
9. 97619544.
10. 739714236.
11. 1098753425.
12. 3432654.
13. 56480640.
14. 33542003.
15. 723669600.
16. 230009090.
17. 1205932.
18. 1329335.
19. 5483452.
20. 70360040.
21. 2202933468.
22. 43449538.
23. 332902033.
24. 192643362240.
25. 39738854302.
26. 39492226288.
27. 1581121276145307.
28. 124892312635200.
29. 80970556097063672.
30. 74070051394414192.
31. 8786880.
32. 978852.
33. 6822504.
34. 4420.
35. 3231360.

Exercise 9.

1. £639, 12s. 7½d.
   £1066, 1s. 0½d.
   £1492, 9s. 5½d.
   £1918, 17s. 10½d.
   £852, 16s. 10d.
2. £42170, 19s. 0¾d.
   £24097, 13s. 9d.
   £36146, 10s. 7½d.
   £48195, 7s. 6d.
   £60244, 4s. 4½d.
3. £6829, 0s. 4d.
   £7682, 12s. 10½d.
   £8536, 5s. 5d.
   £9389, 17s. 11½d.
   £10243, 10s. 6d.
4. £6055, 0s. 7½d.
   £9082, 10s. 11½d.
   £24220, 2s. 6d.
   £15137, 11s. 6¾d.
   £21192, 12s. 2¼d.
   £36330, 3s. 9d.
5. £629, 12s. 1½d.
   £839, 9s. 6d.
   £816, 3s. 1½d.
   £1119, 6s.
   £1888, 16s. 4½d.
6. £30, 19s. 6d.
   £39, 16s. 6d.
   £44, 12s. 4½d.
   £23, 4s. 7½d.
   £19, 18s. 3d.
7. £63, 15s. 2d.
   £72, 17s. 4d.
   £55, 15s. 9½d.
   £47, 16s. 4½d.
   £56, 18s. 6½d.
8. £17, 3s. 5d.
   £20, 9s. 5½d.
   £31, 0s. 9½d.
   £54, 3s. 1d.
   £45, 11s. 4½d.
9. £13, 4s. 8½d.
   £14, 2s. 2½d.
   £10, 3s. 1½d.
10. £2395, 0s. 4d.
    £182172, 17s.
    £1215940, 17s. 4d.
11. £1, 19s. 8d.
12. £46, 14s. 6d.
13. £61343, 15s.
14. £957, 1s. 10½d.
15. £154, 18s. 8¼d.

Exercise 10.

1. 43.
2. 418.
3. 642.
4. 237.
5. 413.
6. 701.
7. 63325.
8. 59411.
9. 157925.
10. 692521.
11. 707342.
12. 359818.
13. 3863723.
14. 598735.
15. 30696187.
16. 299599992.
17. 54436.
18. 29815.
19. £169216.
20. 1357059.

Exercise 11.

1. £48, 12s. 2d.
2. £41, 8s. 1d.
3. £3812, 3s. 6¼d.
4. £4992, 13s. 2d.
5. £29337, 2s. 10d.
6. £2748, 11s. 3¾d.
7. £494985, 1s. 3¾d.
8. £39209, 5s. 8¼d.
9. £2009006, 19s. 7¾d.
10. £99185, 2s. 8½d.
11. £434696, 4s. 11¾d.
12. £2039, 10s. 5⅚d.
13. £2979, 1s. 11¾d.
14. £910, 0s. 11¼d.
15. £16, 19s. 11¾d.
16. £120, 12s. 11¾d.
17. 15s. 3½d.
18. 9s. 11½d.
19. £8, 2s. 9¼d.
20. £399, 10s. 1d.
21. £65, 16s. 7½d.
22. £599, 7s. 6d.
23. £69, 10s.
24. £77, 3s. 4d.
25. £33, 12s. 6d.
26. £63, 10s. 3d.
27. £66, 9s. 11d.
28. £732, 19s. 7d.

Exercise 12.

1. 312.
2. 1841.
3. 3152.
4. 26215.
5. 18117.
6. 22308.
7. 15071.
8. 103301
9. 34079.
10. 85546.
11. 3366.
12. 36418.
13. 954640 + 8.
14. 118670.
15. 3224550.
16. 359736 + 6.
17. 3896536 + 7.
18. 625765 + 3.
19. 17590022 + 3.
20. 1236024 + 3.
21. 151663 + 5.
22. 3473754 + 3.
23. 7513561 + 2.
24. 68929419 + 7.
25. 941901419 + 8.
26. 17571964 + 11.
27. 968289924 + 1.
28. 3134916.
29. 16719850 + 5.
30. 67971921 + 10.
31. 10956.
32. 30263.
33. 918.
34. 35.
35. 144.

Exercise 13.

1. 4s. 1¼d.
2. 2s. 10½d.
3. 2s. 2½d.
4. 6s. 5½d.
5. 4s. 5d.
6. 1s. 3¼d.
7. 18s. 1d.
8. £6, 9s. 5d.
9. £19, 8s. 7½d.
10. £53, 0s. 4¼d.
11. £250, 11s. 9¼d.
12. £1147, 16s. 2d.
13. £1014, 4s. 3½d. +
14. £62, 7s. 8½d. +
15. £375, 5s. 8¾d. +
16. £9002, 14s. 7d.
17. £13289, 0s. 0½d. +
18. £22289, 0s. 4d.
19. £1133, 10s.
20. £45326, 9s. 5¼d. +
21. £36646, 1s. 1d. +
22. £20, 3s. 1½d.
23. £301, 1s. 4¼d.
£3913, 17s. 7¼d.
24. £26, 2s. 0¼d.
25. £150, 6s. 6d.
26. £360, 10s. 4d.

Exercise 14.

1. 29537.
2. 19217 + 16.
3. 73641.
4. 20715.
5. 87304.
6. 4200706.
7. 819070.
8. 32400070.
9. 9909837.
10. 11071039 + 7.
11. 82015925.
12. 4172 + 206.
13. 9281.
14. 5263.
15. 700638.
16. 603709.
17. 2800317.
18. 60050.
19. 635348 + 4011.
20. 16722 + 1570.
21. 28789 + 221.
22. 268256 + 21332.
23. 1511 + 4103.
24. 8019.
25. 69809 + 5163.
26. 306170.
27. 9870690.
28. 1360950.
29. 210105.
30. 69000.
31. 44432 + 44437.
32. 15097 + 322081.
33. 3817 + 597100.
34. 815.
35. 405.
36. 73 hours.
37. 37.
38. 535.
39. 85.
40. 115.
41. 207 pence.
42. 1207.
43. 6028 and 4290 over.
44. 38227; 7 times.
45. 823680.
46. 21.

Exercise 15.

1. £23, 19s. 5½d. +
2. £407, 5s. 4d.
3. £3005, 12s. 4d.
4. £7, 6s. 1½d. +
5. £2607, 1s. 7½d. +
6. £3501, 0s. 10½d.
7. £35, 19s. 5¾d. +
8. £72, 3s. 2¼d.
9. £10, 0s. 4½d.
10. 4s. 7¼d.
11. 16s. 1¾d.
12. £20, 17s. 0¼d.
13. £61, 1s. 3½d.
14. £17, 0s. 8¼d.
15. £2001, 10s. 10¾d.
16. 17s. 11½d.
17. £100, 19s. 11¼d.
18. £17, 12s. 5½d. +
19. £40, 0s. 7¼d.
20. 14s. 2¾d.
21. £300, 12s. 6¼d.
22. £249, 0s. 5d. +
23. £247, 4s. 2¼d. +
24. £13, 0s. 6¼d. +
25. 15s. 4¾d. +
26. £1, 6s. 7d. +
27. £101, 17s. 4½d. +
28. £38, 3s. 1¾d. +
29. £152, 4s. 3d. +
30. £632, 18s. 3d. +
31. 1968.
32. 756.
33. 4560.
34. £1301, 14s. 10d.
35. £83, 5s. 3¾d.; 10 yrs.
36. £3, 2s. 7½d.
37. £7, 2s.

Exercise 16.

1. 750 d.
   24087 d.
   16038 f.
   433 h. d.
   24000 f.
   50904 h. d.
2. 171; 2052.
3. 1488; 11160.
4. 860.
5. 315.
6. £11, 5s. 3d.
   £85, 9s. 9¾d.
7. 2587 in.
   3120 f.
   459000 sec.
   29147 lb.
   387280 dr.
   273 sixpences.
8. 111420 in.
   17262720 sq. in.
   37461703 sq. in.
   7978176 cub. in.
9. 17 wk. 157 hr.
   4 hr. 2600 sec.
   3276 mo. 1 wk. 3 days.
10. 10 lea. 1 m. 4 f. 20 p.
    5 yd. 1 ft. 8 in.
    11 cwt. 3 qr. 1 lb. 1 oz.
    1756 lb. 1 oz.
11. 1000.
12. 3 pints less.
13. 107520; £56.
14. 7344.
15. 6 wk. 3 days.
16. 202 ells 1 qr. 4½ in.
17. 3s. 8¾d.
18. £3109.

Exercise 17.

1. £100709, 13s. 8¼d.
2. 890 oz. 19 dwt. 12 gr.
3. 6571 oz. 16 gr.
4. 7374 tons 12 cwt. 2 qr.
5. 20215 tons 4 cwt. 75 lb. 8 oz.
6. 1071438 sec.
7. 316 m. 5 f. 18 po. 2 yd.
8. 2060 yd. 2 ft. 8 in.
9. 1058 ac. 1 po.
10. 2910 ac. 1 rd. 7 po. 24½ yd.
11. 2962 lb. 8 oz. 5 dwt. 19 gr.
12. 24357 tons 7 cwt. 1 qr. 4 lb. 14 oz.
13. 2597 l. 1 f. 21 yd. 1 ft. 1 in.
14. 95317 sq. yd. 2 sq. ft. 97 sq. in.
15. 23 yr. 6 mo.
16. 45 lb. 7 oz. 11 dr.
17. 43 quarts.

Exercise 18.

1. £158847, 2s. 5¼d.
   £1129579, 10s. 8d.
   £229445, 16s. 10¼d.
   £1041331, 2s. 7¾d.
   £1782617, 14s. 0¼d.
   £3123993, 11s. 7½d.
   £53054941, 16s. 9d.
   £307157402, 15s. 10½d.
2. 217 tons 2 cwt. 2 qr. 10 lb. 1 oz.
   128 tons 13 cwt. 1 lb. 5 oz.
   660 tons 17 cwt. 22 lb. 12 oz.
   1672 tons 2 cwt. 17 lb. 1 oz.
   1982 tons 11 cwt. 2 qr. 12 lb. 4 oz.
   8224 tons.
   33355 tons 2 qr. 24 lb. 11 oz.
3. 357 lb. 5 oz. 16 dwt. 5 gr.
   522 lb. 5 oz. 14 dwt. 11 gr.
   797 lb. 5 oz. 11 dwt. 13 gr.

742 lb. 5 oz. 12 dwt. 3 gr.
2639 lb. 10 oz. 12 dwt.
2941 lb. 11 oz. 8 dwt. 19 gr.
19331 lb. 7 oz. 14 dwt. 23 gr.
66217 lb. 17 dwt. 16 gr.

4. 311 yd. 1 qr. 2 nl.
376 yd. 2 nl.
424 yd.
119 yd. 1 qr.
3672 yd.
26904 yd. 2 nl.

5. 495 wk. 5 d. 13 hr. 30 min.
24982 wk. 3 d. 18 hr. 55 min.
120533 wk. 1 d. 7 hr. 20 min.
2205707 wk. 5 d. 3 hr. 15 min.

6. 1653 qr. 6 bush.
1860 qr. 3 bush. 3 pk.
1492 qr. 7 bush. 3 pk.
1952 qr. 2 bush. 3 pk.
8957 qr. 6 bush. 2 pk.
16215 qr. 7 bush. 2 pk.
116681 qr. 2 bush.

7. 756 gal.
850 gal. 2 qt.
952 gal. 3 qt. 1 pt.
2126486 gal. 6 qt.
126055 gal. 1 pt.

8. 221 ac. 16 po.
400 ac. 2 r. 39 po.
317 ac. 3 r. 13 po.
2344 ac. 2 r. 13 po.
32322 ac. 9 po.
37218 ac. 2 r.

9. 172 sq. yd. 8 ft. 66 in.
13956 sq. yd. 2 ft. 27 in.
247459 sq. yd. 5 ft. 33 in.

10. 334024 c. yd. 7 c. ft. 317 c. in.
5849279 c. yd. 15 c. ft. 1305 c. in.

11. 1068 l. 6 f. 21 po. 4½ yd.
338 l. 6 f. 34 po. 5 yd.
49852 l. 2 m. 1 f. 18 po. 1 yd.

12. 46 tons 4 cwt. 12 lb.

13. 407 yd. 11 in.

14. 266 cwt. 3 qr.; £16, 13s. 5½d.

15. 8812 yd. 3 qr. 2 nl.

16. 834 r. 32 po. 10 yd. 3 ft. 54 in.

17. 167 tons 16 cwt. 15 lb.
273 tons 4 cwt. 3 qr. 14 lb.

Exercise 19.

1. £62, 10s. 8d.
2. £593, 0s. 11½d.
3. 128 yd. 8 in.
4. 43 tons 8 cwt. 1 qr. 12 lb.
5. 110 lb. 6 oz. 12 dwt. 18 gr.
6. 737830 cub. in.
7. 139013 sq. in.
8. 7 ac. 3 r. 26 po. 29½ sq. yd.
9. 227325 oz.
10. 82 days 2 hr. 42 m. 48 sec.
11. £11, 17s.
12. 21 cub. yd. 13 ft. 409 in.
2099929 cub. in.
13. 165 yd. 8 nl. 1¾ in.
14. 137 qr. 2 pk.
15. 77 tons 3 cwt. 111 lb.
72 tons 6 cwt. 111 lb.
16. (*a.*) 31 tons 8 cwt. 3 qr. 11 lb. 13 oz.
(*b.*) 507 lb. 13 oz. 2 dr.
(*c.*) 69769 lb. 4 oz. 11 dwt. 20 gr.
(*d.*) 41 qr. 2 bush.
(*e.*) 64475 days 1362 min. 56 sec.
(*f.*) 129 ac. 2 r. 38 po. 27 yd. 1 ft. 12 in.
(*g.*) 1541 ells 1 qr. ½ in.

Exercise 20.

1. £2516, 5s. $9\frac{3}{4}$d. + 6.
   £1957, 2s. $3\frac{3}{4}$d.
   £550, 8s. $9\frac{1}{4}$d. + 7.
   £270, 19s. $8\frac{1}{4}$d. + 54.
   £202, 9s. $2\frac{1}{4}$d. + 72.
2. 19324 yd. 2 ft. 7 in.
   8282 yd. 3 in.
   11594 yd. 2 ft. 9 in.
   5270 yd. 1 ft. 3 in.
   4459 yd. 1 ft. 9 in.
3. 19717 tons 12 cwt. 26 lb. $7\frac{1}{9}$ oz.
   29576 tons 8 cwt. 39 lb. $10\frac{2}{3}$ oz.
   25351 tons 4 cwt. 34 lb.
   13650 tons 13 cwt. 9 lb. $11\frac{1}{13}$ oz.
   4328 tons 5 cwt. 14 lb.
4. 3868 ac. 3 r. 3 po. 28 sq. yd. 5 ft. $113\frac{1}{7}$ in.
   5416 ac. 1 r. 5 po. 15 sq. yd. 8 ft. $14\frac{2}{5}$ in.
   873 ac. 2 r. 15 po. 2 sq. yd. 5 ft. $11\frac{19}{31}$ in.
   933 ac. 3 r. 14 po. 22 sq. yd. 5 ft. $6\frac{6}{29}$ in.
   44 ac. 3 r. 2 po. 7 sq. ft. $134\frac{182}{605}$ in.
5. 64 gal. 1 qt. $2\frac{10}{79}$ gills.
   118 gal. 1 pt. $1\frac{4}{93}$ gills.
   81 gal. 3 qt. 1 pt. $2\frac{14}{31}$ gills.
   2 qt. $2\frac{430}{9009}$ gills.
6. 16 days 7 hr. 22 min. $26\frac{562}{703}$ sec.
   224 days 18 hr. 49 min. $24\frac{12}{17}$ sec.
   184 days 21 hr. 40 min. $38\frac{22}{31}$ sec.
   136 days 11 hr. 25 min. $42\frac{6}{7}$ sec.
7. 26 Eng. ells 3 qr. 1 nl. $1\frac{7}{50}$ in.
   22 Eng. ells 2 qr. 1 nl. $1\frac{28}{89}$ in.
   6 Eng. ells 2 qr. 2 nl. $0\frac{927}{1228}$ in.
8. 14 oz. 14 dwt. 10 gr.
9. 108 sq. in.
10. 2s. 3d.
11. 2 tons 15 cwt. 3 qr. 20 lb. $6\frac{60}{59}$ oz.
12. 1 ac. 1 r. 14 po. 22 sq. yd. 2 ft. $87\frac{3}{19}$ in.
13. 73.
14. 108 days.
15. 7 po. 5 yd. 1 ft. $3\frac{158}{227}$ in.
16. 47.
17. 7041 times.
18. 91.
19. 309.
20. 77.

Exercise 21.

1. 945.
2. 27.
3. 507.
4. £297, 3s.
5. 1982880.
6. 476 cwt.
7. 52 tons.
8. 2450.
9. £24, 0s. $4\frac{1}{2}$d.
10. 119.
11. 3276 sq. in.
12. 40 yd.
13. 552.
14. 33048.
15. 5 ft.
16. 10.

Exercise 22.

1. 1; 7; 11; 23; 1; 7.
2. 364; 1235; 24336; 22848; 1417780; 162540.
3. 3; 3855.
4. 31; 59; 81; 13; 41.

Exercise 23.

1. $\frac{15}{11}$; $\frac{37}{5}$; $\frac{31}{5}$; $\frac{20315}{201}$; $\frac{42505}{707}$.

2. $1\frac{9}{11}$; $3\frac{5}{16}$; $114\frac{3}{7}$; $73\frac{12}{13}$; $29\frac{77}{171}$.

3. $\frac{135}{315}$, $\frac{140}{35}$, $\frac{54}{315}$; $\frac{180}{396}$, $\frac{33}{396}$, $\frac{140}{396}$; $\frac{3264}{3648}$, $\frac{3591}{3648}$, $\frac{1976}{3648}$; $\frac{8262}{13770}$, $\frac{4860}{13770}$, $\frac{2210}{13770}$, $\frac{9855}{13770}$; $\frac{2425032}{16480800}$, $\frac{9919000}{16480800}$, $\frac{15224139}{16480800}$, $\frac{7190400}{16480800}$.

4. $\frac{8008}{24640}$, $\frac{11264}{24640}$, $\frac{24080}{24640}$.

5. $\frac{84150}{89100}$, $\frac{46200}{89100}$, $\frac{113300}{89100}$, $\frac{30294}{89100}$, $\frac{16533}{89100}$.

6. 84; 287.

7. $2\frac{1}{3}$; $1\frac{3}{5}$; $3\frac{2}{11}$; $3\frac{15}{48}$; $\frac{5}{17}$; $\frac{8}{105}$; $\frac{4}{203}$; $\frac{13}{152}$; $\frac{3}{476}$.

8. $3\frac{1}{2}$; $21\frac{1}{7}$; $92\frac{7}{16}$; $530\frac{2}{3}$.

9. $\frac{5}{16}$; $\frac{69}{104}$; $4\frac{13}{28}$; $1\frac{1478}{3053}$.

10. $\frac{3}{5}$; $\frac{16}{19}$; $\frac{50}{201}$.

11. $4\frac{7}{9}$; 5; $2\frac{13}{18}$; $4\frac{1}{15}$; $47\frac{4}{17}$; $3\frac{50}{151}$.

Exercise 24.

1. $1\frac{4}{5}$; $2\frac{18}{19}$; $8\frac{1}{7}$; $25\frac{1}{4}$; $9\frac{37}{41}$; $36\frac{33}{37}$; $4\frac{9}{48}$; $7\frac{16}{71}$; $30\frac{3}{80}$; $44\frac{62}{99}$.

2. $1\frac{25}{84}$; $2\frac{347}{3960}$; $2\frac{699}{1040}$; $2\frac{14177}{58520}$; $15\frac{7}{36}$; $9\frac{203}{270}$; $12\frac{61}{9900}$; $334\frac{191717}{240240}$.

3. $26\frac{21}{160}$; $49\frac{535}{1008}$.

4. $\frac{5}{9}$; $\frac{4}{13}$; $\frac{3}{11}$; $\frac{5}{7}$; $\frac{3}{13}$; $\frac{1}{18}$; $\frac{4}{5}$; $\frac{9}{11}$; $\frac{1}{91}$; $\frac{1}{3}$; $\frac{62}{273}$.

5. $2\frac{3}{44}$; $8\frac{22}{45}$; $6\frac{730}{1377}$; $4\frac{4}{5}$; $2\frac{145}{156}$; $2\frac{1}{7}$; $4\frac{83}{95}$; $7\frac{411}{584}$; $\frac{63}{2178}$; $1\frac{5786}{2453}$; $\frac{2237}{6265}$; $\frac{77}{19380}$.

6. $11451\frac{3}{17}$.

7. $\frac{6}{65}$; $\frac{64}{285}$; $1\frac{1}{11}$; $3\frac{33}{56}$; 30; $287\frac{1}{9}$.

8. $\frac{14527}{28224}$.

9. $55\frac{9}{10}$.

10. $1027\frac{458}{1045}$.

11. $95\frac{13}{16}$; $\frac{152}{315}$; $2\frac{1}{7}$; $2\frac{74}{91}$; $\frac{91}{300}$; $1\frac{54}{269}$; $\frac{32}{217}$; $\frac{209}{444}$.

12. $\frac{931}{2070}$.

13. $1\frac{505}{916}$.

14. $15\frac{3}{67}$.

15. $145\frac{1}{4}$.

Exercise 25.

1. $\frac{20}{49}$; $1\frac{31}{33}$; $\frac{47}{252}$; $\frac{10}{13}$; $1\frac{133}{169}$.

2. $\frac{1}{30}$.

3. $\frac{71}{894}$.

4. 113 tons 8 cwt. 2 qr.

5. $27\frac{157}{160}$.

6. £289, 4s.

7. 132 lb. 2 oz. 13 dwt. 1 gr.

8. £5600, 3s. $9\frac{1}{6}$d.

9. $5\frac{4643}{7980}$.

10. $\frac{24185}{1504}$.

11. 32528.

12. $59\frac{23}{28}$; $314\frac{326}{1001}$; $689\frac{29}{44}$; $\frac{781}{5000}$.

13. £1049, 1s. $1\frac{5}{7}$d.

14. £523, 15s. 10d.

15. 71 ac. 2 r. $3\frac{3}{7}$ poles.

16. 23 tons 5 cwt. 1 qr. 21 lb.; 10 tons 11 cwt. 2 qr. 7 lb.

17. £74, 6s. $6\frac{6}{13}$d.

18. 538 tons 13 cwt.; 199 tons 10 cwt.; 513 tons.

19. £121, 4s. 7d.; £55, 2s. 1d.; £11, 10s.; £54, 12s. 6d.

Exercise 26.

1. ·017; ·016; 337·4; 20902·49; 164·028; 305·805; 31003000·0205; 62830048·1537.

2. $\frac{19}{20}$; $30\frac{17}{20}$; $27\frac{81}{1000}$; $1007\frac{3}{200}$; $\frac{1}{400}$; $8\frac{97}{160}$.

3. ·1875; 1·3375; 2·73828125; ·416; 19·82.

4. 3167, 316700, 31670000; ·00039816, ·000039816, ·00000039816.

5. 11643·61525; 77530·00996; 29466·195447.

6. 714·57184.

7. 197·05.

8. 4·043.

9. 686·692.

10. 231·35625; 378·$\dot{3}\dot{6}$; 49·6112; ·189875; 1·92962; 607524393; 14866884·97788.

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